Universidad
Carlos III de Madrid
www.uc3m.es

TESIS DOCTORAL

Essays on Fiscal Policy

Autor:

Meng Li

Director/es:

Matthias Kredler

DEPARTAMENTO DE ECONOMÍA

Getafe, abril de 2017

Universidad Carlos III de Madrid
www.uc3m.es

TESIS DOCTORAL

Essays on Fiscal Policy

Autor: *Meng Li*

Director/es: **Matthias Kredler**

Firma del Tribunal Calificador:

Firma

Presidente:

Vocal:

Secretario:

Calificación:

Getafe, de de

Acknowledgements

I express my deep gratitude to my supervisor, Matthias Kredler, and also to Andrés Erosa, Pedro Gomes, Hernán Seoane, Manuel Santos, Salvador Ortigueira, Antonia Díaz. Their selfless help and guidance will be important in my whole life, not only as an economist, but also as a person.

To complete this dissertation, I also listened to suggestions, critics and comments from many other researchers, such as Fabrizio Zilibotti, Timothy Kehoe, Karl Walentin, Martín Gonzalez Eiras, Stefan Niemann, Alberto Martín, Nobuhiro Kiyotaki, Selahattin Imrohoroglu. Their precious opinions help improve the thesis. I give my thanks for all their help.

I want to acknowledge my colleagues Alessandro Peri, Sebastian Panthöfer, Federico Masera, Salvatore Lo Bello, Andrés García, Christos Mavridis, Mehdi Hamidi Sahneh, Federico Curci, Mian Huang, José Manuel Carbó Martínez, Beatriz González and Tomas Martínez. Your inspiration in all aspects of economics broadens my horizon. I give special thanks to Omar Rachedi, who functioned as my second supervisor for a long time. Your critics improve my research and economic thoughts.

I also thank all secretaries working in the department of economics, organizers of student seminars in UC3M and financial support from Spanish government.

At last, I would like to thank Rui Cui with whom I have spent a lot of happy time. Funny anecdotes and jokes shared between us are the most valuable legacy, apart from the dissertation, in my PhD life.

Abstract

Macroeconomics has paid profound attention to policy studies over the last decades. My PhD thesis discusses how to assess fiscal policy over the business cycle in unconventional but justifiable environments - either with nonstandard preferences or with less-studied idiosyncratic risks.

Chapter 1 justifies policy interventions with exotic preferences. I study a production economy with preferences featuring loss aversion, a core concept commonly accepted in behavioral economics. The representative household obtains utility directly from fluctuations of asset returns, in addition to consumption. I ask whether loss aversion affects equilibrium conditions, whether equilibrium is efficient, and what Ramsey optimal fiscal policy is. I show analytically that the more loss aversion and the more concerns on the psychological utility, the less investment in risky assets in the risky steady state of equilibrium. Numerical results show that capital stock, consumption and output decrease as loss aversion parameters increase. I uncover that the competitive equilibrium is inefficient as long as the agent is loss averse due to pecuniary externality. The household does not internalize the price effect on her welfare so that she invests more in capital than the optimum requires. The Ramsey allocation rationalizes policy interventions: the government should tax capital accumulation to reduce capital stock and raise equity premium.

Chapter 2 studies the effect of fiscal policy on investment and the welfare of heterogeneous agents over the business cycle considering independent and identically distributed idiosyncratic investment risks whose volatility is assumed to be countercyclical in line with data. All entrepreneurs make identical saving and portfolio choices each period, allowing for exact aggregation which facilitates computation. The model matches income inequality and dynamics of the income distribution over the cycle in the US data. The government sets rules of capital income tax rate and debt as functions of current output. I calibrate the cyclicality of fiscal policy to the US data as the baseline and adjust the

parameters indicating cyclicality to study the effect of counterfactual policy. Both capital tax and debt policy create welfare conflicts between entrepreneurs and workers. The policy that optimizes utilitarian social utility specifies that the capital income tax rate should increase by 0.45 percentage point and the debt should increase by 0.37% facing a 1% decrease in output. It is possible that the government should reduce the capital tax in the recession if it increases the weight of workers on social welfare. The impulse response of aggregate variables to a negative productivity shock indicates that in general, the higher capital tax rate and the less debt when the adverse shock hits, the higher capital and output in the early stage after the shock. The result of the welfare conflict is robust to a constant labor tax and a varying consumption tax, but not to the removal of countercyclicality of idiosyncratic investment risks or to no idiosyncratic risks.

Contents

List of Figures

List of Tables

Chapter 1

Loss Aversion, Inefficiency and Policy Interventions

1.1 Introduction

Macroeconomists have paid profound attention to the behavior of economic agents over the business cycle and policy analysis related to it. However, an assumption pervades the majority of studies that economic agents care only about allocations, such as individual consumption, leisure and public goods. This convention possibly omits some components which may play a role in the welfare. Results from economic experiments suggest that people feel excited if they gain in the investment; in addition, a loss affects a person more than the same amount of gain[1]. This phenomenon is named as "loss aversion".

Kahneman and Tversky (1979) propose loss aversion as a part of prospect theory. Loss aversion postulates that economic agents evaluate decisions based on a reference point, which implies that utility can be generated from not only the absolute value, but also the change of assets. Besides, economic agents value gains differently from the way in which they value losses as experimental evidence shows. They obtain a greater disutility from a loss than a utility from the same amount of gain. If we use the utility function representation,

$$-v(-x) > v(x), \text{ when } x > 0; \text{ and } v(0) = 0,$$

where x indicates the gain from investment and v denotes the loss aversion utility from gain.

[1] Such an example can be found in Kahneman and Tversky (1979)

Loss aversion has been confirmed not only from experiments, but also by empirical studies focusing on certain industries. For instance, Pope and Schweitzer (2011) test for the presence of loss aversion using a large sample of professional golfers' performance on the PGA Tour. They verify that even the best golfers show loss aversion. Camerer et al. (1997) use data on cabdrivers in New York City to reveal that drivers are afraid of falling below a target income, consistent with loss aversion. The driver decides working hours of a day largely depending on the comparison between actual daily income and the target: the driver stops sooner if he gets the target income more quickly; furthermore, earning less than the target shows more effect than the same amount of earning more than the same target.

Loss aversion is often employed in finance and behavioral economics. For example, Benartzi and Thaler (1995) apply loss aversion to explain the equity premium puzzle. They focus on a certain assets market and claim that a reasonable loss aversion degree generates the equity premium if agents check their account once a year. Ang et al. (2006) show that agents place greater weight on downside risk, indicating loss aversion in individual investors. Ang et al. (2006) recover that large institutional investors exhibit loss aversion by matching investment behaviors with prospect theory.

Macroeconomics still rarely adopts loss aversion. A model without loss aversion may misunderstand agents' behaviors and lead to an inefficient policy. So I ask whether a model with loss aversion behaves differently in the competitive equilibrium. I also investigate the efficiency of competitive equilibrium and derive Ramsey optimal fiscal policy.

I embed loss aversion in a business cycle model. Besides consumption, a loss averse household obtains utility from expected gains from risky assets relative to returns to riskfree assets. I characterize the competitive equilibrium and discuss analytically how capital stock in equilibrium evolves with loss aversion degree indicating how much a loss affects the welfare relative to a gain, and with relative weight of loss aversion utility showing the concerns on direct impact of fluctuations of asset prices on the welfare. I show that the more loss aversion and the more concerns on the loss aversion utility, the less investment in risky assets in the risky steady state[2] equilibrium.

I show analytically that the competitive equilibrium is inefficient by considering a constrained optimality problem in a two-period model following Davila et al. (2012).

[2]The risky steady state is the point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is 0 at this date.

The atomistic household takes prices as given and does not internalize the influence of her choice on prices. With loss aversion preferences, prices directly affect the utility so that pecuniary externality creates a gap between the equilibrium and the optimum even without any idiosyncratic risks and frictions. Then I construct a social planner's problem in which the planner has identical preferences with the household and maximize the household's welfare subject to feasibility constraint. Since prices appear in the preferences, I employ the method by Bianchi and Mendoza (2013) in which the social planner assigns capital stock for the representative agent and keeps the formation of market prices. The social optimum differs from the competitive equilibrium as long as the agent is loss averse in the characterization.

I carry out quantitative analysis. I plot the risky steady state as a function of loss aversion parameters. The numerical result shows that an increase in either loss aversion parameter provokes a drop in capital stock, output and consumption. Given asset prices, stronger fear for loss causes the household to invest less on capital. Then less capital input results in less output and consumption. I depict the constrained optimum and the competitive equilibrium as loss aversion parameters vary to confirm that the equilibrium deviates from the optimum. The cost of fluctuations over the business cycle in my model, measured by consumption equivalent variation, reaches more than 5%, which is much more than many previous estimations. This is because the household directly obtains disutility from potential losses from investment in addition to more volatile consumption.

I wonder whether policy interventions can correct the inefficiency of competitive equilibrium. I add into the model a government sector which finances its spending by a lump-sum tax and a distortionary capital income tax and explore the Ramsey fiscal policy. Judd (1985) and Chamley (1986) demonstrate that the optimal capital income tax rate tends to zero in the long run, which has been confirmed when relaxing a number of assumptions. Chari et al. (1994) study the optimal policy in the business cycle quantitatively and show that the long-run mean of capital tax rate is close to zero even with a relatively high risk aversion. In my model, the Ramsey policy requires a relatively high capital income tax (about 65%). Simultaneously, the government should levy a low lump-sum tax to balance the budget. The impulse response to a negative productivity shock generates two different qualitative behaviors of policy instruments. Without loss aversion, both the capital income tax rate and the lump-sum tax remain constant after the shock. With loss aversion, they move dramatically in an opposite direction: the tax rate increases while the lump-sum tax drops.

This paper is in line with a great literature of business cycle and policy analysis, especially works on fiscal policy over the business cycle. In addition to what I have mentioned above, for example, Aiyagari et al. (2002) study the optimum quantity of debt based on a model of a large number of infinitely-lived households whose saving behavior is influenced by precautionary saving motives and borrowing constraints. They find that the welfare gains from reaching the optimum level of debt are small. Schmitt-Grohé and Uribe (2004), Schmitt-Grohé and Uribe (2004), Chugh (2006) and Arseneau and Chugh (2012) address the optimal policy issues by considering imperfect competition, sticky prices, sticky wages and sticky prices together, and frictional labor markets over the cycle, respectively. Nevertheless, none of them incorporate exotic preferences.

This paper belongs to macroeconomic studies that use exotic preferences in a business cycle model. Angeletos and Calvet (2006) and Angeletos (2007) both apply Epstein-Zin preferences to see the effect of idiosyncratic production risks on the equilibrium over the business cycle and growth. Epstein-Zin preferences, by differentiating the elasticity of intertemporal substituion from risk aversion, help to figure out that the underlying factor lies in the elasticity of intertemporal substitution. Croce et al. (2012) investigate the optimal fiscal policy which functions through the channel of asset prices. They also use Epstein-Zin preferences to generate a realistic equity premium. Chugh (2007) derives Ramsey fiscal and monetary policies with habit formation. Habit persistence partly predicts highly persistent inflation. But above research never applies loss aversion preferences that feature asymmetry in the effect of gains and losses and discontinuous at the reference point.

The closest studies to my paper are from a growing literature that uses loss aversion in the general equilibrium. Barberis et al. (2001) study asset pricing considering loss aversion in financial wealth and discover that their framework can explain the high mean, excess volatility and predictability of stock returns. Barberis and Huang (2001), and Berkelaar and Kouwenberg (2009) explore equilibrium firm-level stock returns with loss aversion in two different economies. Andries (2012) and Pagel (2015) readdress asset pricing with loss aversion. Pagel (2013) uses the expectation-based reference-dependent preference featuring loss aversion to explain empirical observations about life-cycle consumption.

Ahrens et al. (2017) develop a theory under loss aversion which successfully explains why prices are more sluggish upwards than downwards in response to temporary demand shocks, while they are more sluggish downwards than upwards in response to permanent demand shocks as empirical evidence finds. Lepetyuk and Stoltenberg (2013) reconcile the changes in consumption inequality in the data in response to the increase in income inequality with loss aversion preferences. Yet none of these papers construct models in a production economy; in addition, government policies are not involved in these settings. Thus I consider that this paper is a novel attempt in the theoretical aspect.

The paper is organized as follows. I develop a private economy model with loss aversion in Section 2 and determine the competitive equilibrium. I also analyze the steady state of this economy. In Section 3, I show the inefficiency of competitive equilibrium and characterize the constrained optimum. Section 4 is devoted to the numerical analysis. Section 5 formulates a model with a government and obtains the Ramsey policy. Section 6 concludes.

1.2 The Model with Loss Aversion

This section presents a real business cycle model with preferences encompassing consumption and utility from shifts in asset returns. The latter features loss aversion. To model the investment choice of a loss averse household, two assets are traded in asset markets: one is risky while the other is riskfree. I characterize the competitive equilibrium. I find that loss aversion does not affect the deterministic steady state but does influence the risky steady state by studying comparative statics.

1.2.1 Economy

The economy is populated by an infinitely-lived representative household that is endowed with one unit of time in each period. The household maximizes expected lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t,\ \beta \in (0,1). \tag{1.1}$$

A representative firm produces a single consumption good with labor n_t and capital k_t. The total output Y_t is consumed or used to augment capital stock. The feasibility constraint is

$$c_t + k_{t+1} = Y_t + (1 - \delta)k_t, \tag{1.2}$$

where $\delta \in (0, 1)$ is the depreciation rate.

I assume that the production function, $Y_t = Z_t F(k_t, n_t)$, has constant returns to scale and that it is increasing and concave in each argument. Exogenous aggregate productivity, Z_t, follows an AR(1) process,

$$\ln Z_{t+1} = \rho_z \ln Z_t + \sigma_z \epsilon^z_{t+1}, \tag{1.3}$$

where ρ_z denotes the autoregressive parameter for the moving process of productivity, σ_z represents the standard deviation of one-time technological innovation and the innovation, ϵ^z_{t+1}, is independently distributed as a standard normal for any $t \geq 0$.

Product and factor markets are assumed to be competitive.

1.2.2 Firm

The firm takes as given the wage rate, w_t, and the rental rate, r_t, hires labor and rents capital from the household, produces final consumption goods and maximizes its profit,

$$\Pi_t = Z_t F(k_t, n_t) - r_t k_t - w_t n_{t.} \tag{1.4}$$

The production follows a constant-returns-to-scale neoclassical technology F, where it is strictly increasing and concave in any input.

1.2.3 Household

The representative household is endowed with some initial capital, k_0. At period t, the household receives income from labor supply, capital rental and non-state-contingent private bonds traded among individuals, and then determines the amount of consumption, capital accumulation and the purchase of next period's individual assets by maximizing (1) subject to following sequences of budget constraints and nonnegativity constraints:

$$c_t + k_{t+1} + a_{t+1} = w_t n_t + r_t k_t + (1 - \delta)k_t + R^f_t a_t, \tag{1.5}$$

$$c_t \geq 0,\ k_t \geq 0,\ 0 \leq n_t \leq 1. \tag{1.6}$$

R_t^f is the gross return of private bonds from $t-1$ to t, depending only on the state at $t-1$, so that individual assets are riskfree. The uncertainty of the return to capital rental arises from unknown productivity shocks. When the household makes investment decisions, she undertakes risks if she augments capital stock while avoids risks if purchasing bonds.

1.2.4 Preference Specification

As the main feature of this paper, I assume that the household enhances her utility, in addition to consumption, if she gains in investment. It implies that the household cares about fluctuations in investment markets independent of total wealth. It reflects the observation that individuals in reality feel excited when they succeed in the capital market. Mathematically, I express the instantaneous utility at t consisting of consumption at the current period, c_t, and expected gains from capital investment next period with respect to a reference point defined later, X_{t+1}, as

$$U_t = u(c_t) + \eta\beta E_t\left[v(X_{t+1})\right],$$

where u is strictly increasing, concave and two times continuously differentiable in c, and v reflects loss aversion utility. η represents the relative weight on the utility from expected gain compared to consumption. The preference returns to the standard model merely containing consumption when $\eta = 0$. I formulate preferences over consumption and expected gain of capital investment in the spirit of Barberis et al. (2001), whose preference specification consists of two additively separable terms: utilities from consumption and expected one-period-after fluctuations in financial asset values. I also consider the scenario when the agent obtains the loss aversion utility from realized gain, that is, $\eta v(X_t)$. The corresponding first-order conditions show that the timing alternative does not affect investment decisions.

In particular,

$$v(X_{t+1}) = \begin{cases} X_{t+1}, & \text{if } X_{t+1} \geq 0; \\ \lambda X_{t+1}, & \text{if } X_{t+1} < 0. \end{cases}$$

The parameter λ denotes the loss aversion degree and it is assumed to be strictly larger than 1, indicating that a certain amount of loss impacts greater in absolute values than the same amount of gain. The functional form of v, a linear function with a kink, is a simplification from the literature of prospect theory. Tversky and Kahneman (1992) suggest the utility function over gains and losses:

$$\tilde{v}(X) = \begin{cases} X^{\gamma}, & \text{if } X_{t+1} \geq 0; \\ -\lambda(-X)^{\gamma}, & \text{if } X_{t+1} < 0. \end{cases}$$

Yet it only improves the quantitative behavior with prospects of only gains or only losses. Loss aversion remains as long as the link exists. For simplicity, I assume a linear form. In addition, I filter out some other distinctive features of prospect theory since I only focus on loss aversion in the production economy. To summarize, my use of $v(X)$ captures a central idea of prospect theory by Kahneman and Tversky (1979) that people care about changes in financial wealth and that they are loss averse over these changes. Some recent studies such as Pagel (2013) and Pagel (2013) model loss aversion with the expected consumption as the reference point. I argue that this modelling way has modified the original prospect theory. My paper still targets on financial wealth instead of consumption in the preference component of loss aversion.

I define the gross returns to capital rental as $R_t^k = r_t + 1 - \delta$. In this paper, X_{t+1} is assumed to have the form:

$$X_{t+1} = k_{t+1}R_{t+1}^k - k_{t+1}R_{t+1}^f.$$

At $t+1$, the household receives $k_{t+1}R_{t+1}^k$ from investment in risky assets. In fact, $k_{t+1}R_{t+1}^k$ is also the total financial wealth in equilibrium. I use the gross return to private bonds as the reference point for the household. $k_{t+1}R_{t+1}^f$ is the opportunity cost of investment in risky assets: suppose that an agent has already invested k_{t+1} in risky assets and expects to earn $k_{t+1}R_{t+1}^k$, yet she would get $k_{t+1}R_{t+1}^f$ if investing in bonds. Then if $R_{t+1}^k > R_{t+1}^f$, it is defined as a gain; if $R_{t+1}^k < R_{t+1}^f$, a loss. Denote $D_{t+1} = R_{t+1}^k - R_{t+1}^f$ as equity premium, so I can interprete X_{t+1} as a product of equity premium and capital stock and rewrite $X_{t+1} = D_{t+1} \cdot k_{t+1}$. Then

$$v(X_{t+1}) = v(D_{t+1}) \cdot k_{t+1}$$

as the result of linear transformation. At t, the agent only has one unknown, $t+1$'s productivity Z_{t+1}. Since the distribution of innovation is common knowledge, the agent computes the next period's expected gain conditional on current information[3].

Since my paper concentrates on how loss aversion influences investment, I simplify the model by an exogenous labor supply in equilibrium. I choose incomplete markets because of several reasons. First, the reference point of loss aversion utility may be unclear in complete markets. An investor could consider various measures as the benchmark of expected gains. Second, incomplete markets better mirror the real world.

1.2.5 Competitive Equilibrium

I define a competitive equilibrium as a stochastic sequence of prices $\{w_t, r_t, R^f_{t+1}\}_{t=0}^{\infty}$, a stochastic sequence of allocations $\{c_t, k_{t+1}, a_{t+1}\}_{t=0}^{\infty}$, such that

(1) Given prices $\{w_t, r_t, R^f_{t+1}\}_{t=0}^{\infty}$, the household maximizes her lifetime utility by choosing $\{c_t, k_{t+1}, a_{t+1}\}_{t=0}^{\infty}$, and the firm maximizes its profit by choosing the amount of inputs.

(2) Bond market clearing: $a_{t+1} = 0$ for all t.

(3) Goods market clearing: feasibility constraint, (1.2), holds.

In equilibrium labor supply is inelastic, $n_t = 1$. Factor prices are determined by solving the representative firm's problem.

$$r_t = Z_t F_k(k_t, 1), \tag{1.7}$$

$$w_t = Z_t F_n(k_t, 1). \tag{1.8}$$

Euler equations below characterize the solution to the household's maximization problem:

[3]Take a simple example: the agent knows that at $t+1$, the bond will be sold at \$0.9; from today's high productivity and the distribution of its evolution process, she calculates that next period the gross return to risky assets will reach \$1.3 with a probability of 80% while may decline to \$0.7 otherwise. Then the total after-one-period expected gain net of the opportunity cost equals to \$ $(0.32 - 0.04\lambda)$.

$$u'(c_t) = \beta E_t \left[R^k_{t+1} u'(c_{t+1})\right] + \eta \beta E_t \left[v(D_{t+1})\right], \tag{1.9}$$

$$u'(c_t) = \beta R^f_{t+1} E_t \left[u'(c_{t+1})\right]. \tag{1.10}$$

Using equilibrium conditions, I rewrite the gross return to capital in period $t+1$, $R^k_{t+1} = Z_{t+1} F_k(k_{t+1}, 1) + 1 - \delta$. By construction, only Z remains unknown in period t. Then if we focus on the loss aversion utility from gain, the individual is indifferent from investing in risky and riskfree assets when the realized value of Z_{t+1}, denoted by z_{t+1}, equals $z^{idf}_{t+1} = \frac{R^f_{t+1} - 1 + \delta}{F_k(k_{t+1}, 1)}$, the solution to the equation $R^k_{t+1} = R^f_{t+1}$. z^{idf}_{t+1} will be larger if the agent accumulates more capital. It is confirmed by differentiating z^{idf}_{t+1},

$$\frac{\mathrm{d}z^{idf}_{t+1}}{\mathrm{d}k_{t+1}} = \frac{F_k(k_{t+1}, 1)\frac{\mathrm{d}R^f_{t+1}}{\mathrm{d}k_{t+1}} - \left(R^f_{t+1} - 1 + \delta\right) F_{kk}(k_{t+1}, 1)}{\left(F_k(k_{t+1}, 1)\right)^2} > 0,$$

where

$$\frac{\mathrm{d}R^f_{t+1}}{\mathrm{d}k_{t+1}} = \frac{-u''(c_t) - \beta R^f_{t+1} E_t \left[u''(c_{t+1}) R^k_{t+1}\right]}{\beta E_t \left[u'(c_{t+1})\right]} > 0.$$

Only if the realization of next period's productivity surpasses this cutoff can the agent gain from her investment. Thus, a large cutoff squeezes the gain possibility, bringing about pessimistic beliefs about the expected payoff. It implies that more capital investment leads to more loss aversion disutility given the same expected productivity.

With the above analysis, I rewrite the expected utility from next period's investment gain conditional on t's information as

$$\begin{aligned}
& E_t \left[v\left(X_{t+1}\right)\right] \\
&= k_{t+1} E_t \left[v\left(D_{t+1}\right)\right] \\
&= k_{t+1} \left[\int_0^{z^{idf}_{t+1}} \lambda \left(R^k_{t+1} - R^f_{t+1}\right) \mathrm{dF}_{Z_{t+1}|Z_t = z_t}(z) + \int_{z^{idf}_{t+1}}^{\infty} \left(R^k_{t+1} - R^f_{t+1}\right) \mathrm{dF}_{Z_{t+1}|Z_t = z_t}(z)\right],
\end{aligned}$$

where $\mathrm{F}_{Z_{t+1}|Z_t = z_t}(z)$ is the conditional cumulative distribution function of next period's productivity, Z_{t+1}, in period t. Simply put, I separate the utility from expected gains from that from expected losses and calculate each conditional expectation. The formulation is motivated by Kőszegi and Rabin (2009), where the reference point of unknown future consumption is defined as continually updated conditional expectations of future consumption in a dynamic environment. Hence this modelling way can be viewed as a special case of their more general setting, in the sense that at a certain period, the reference point is fixed according to the history up to that period with only productivity unsure, while their model embraces both uncertain realizations and uncertain reference points.

Given the stochastic process of productivity, I simplify the expression of $E_t\left[v(D_{t+1})\right]$ in the fashion of certainty equivalence. Appendix B describes the detail of simplification.

$$\begin{aligned} E_t\left[v(D_{t+1})\right] =& F_k(k_{t+1},1) z_t^{\rho} e^{\frac{\sigma_z^2}{2}} \left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho \ln z_t + \sigma_z^2)}{\sigma_z}\right)\right] + \\ &+ (1-\delta-R_{t+1}^f)\left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho \ln z_t}{\sigma_z}\right)\right], \end{aligned}$$

where $\Phi(\cdot)$ represents the cumulative distribution function of standard normal distribution. Technically speaking, to compute the conditional expectation avoids the discussion of kinks around the reference point so that it facilitates further analysis.

I study the comparative statics of parameters on equilibrium allocations by deriving the effect of loss aversion parameters, λ and η, on capital in equilibrium. I differentiate the first Euler equation (1.9) with respect to λ and η, respectively.

$$\begin{aligned} &- u''(c_t)\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda} \\ &= \beta E_t\left[u''(c_{t+1})\left(R_{t+1}^k\right)^2 + u'(c_{t+1}) Z_{t+1} F_{kk}(k_{t+1},1)\right]\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda} + \\ &+ \eta\beta\left\{\int_0^{z_{t+1}^{idf}}\left[\left(R_{t+1}^k - R_{t+1}^f\right) + \lambda\left(Z_{t+1}F_{kk}(k_{t+1},1) - \frac{\mathrm{d}R_{t+1}^f}{\mathrm{d}k_{t+1}}\right)\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda}\right]\mathrm{dF}_{Z_{t+1}|Z_t=z_t}(z) + \right. \\ &+ \int_{z_{t+1}^{idf}}^{\infty}\left(Z_{t+1}F_{kk}(k_{t+1},1) - \frac{\mathrm{d}R_{t+1}^f}{\mathrm{d}k_{t+1}}\right)\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda}\mathrm{dF}_{Z_{t+1}|Z_t=z_t}(z) + \\ &\left. +(\lambda-1)\left(z_{t+1}^{idf}F_k(k_{t+1},1) + 1 - \delta - R_{t+1}^f\right)\frac{\mathrm{d}z_{t+1}^{idf}}{\mathrm{d}k_{t+1}}\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda}\right\}. \end{aligned}$$

$\left(z_{t+1}^{idf}F_k(k_{t+1},1) + 1 - \delta - R_{t+1}^f\right)$ equals 0 after evaluating the value of cutoff z_{t+1}^{idf}, so

I cross out the last line. From the above equation, I obtain

$$
\begin{aligned}
&\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda} \\
&= \eta\beta \int_0^{z_{t+1}^{idf}} \left(R_{t+1}^k - R_{t+1}^f\right) \mathrm{dF}_{Z_{t+1}|Z_t=z_t}(z) \div \Big\{-\beta E_t \Big[u''(c_{t+1}) \left(R_{t+1}^k\right)^2 + u'(c_{t+1}) Z_{t+1} \\
&F_{kk}(k_{t+1},1)\Big] - \eta\beta \left\{ F_{kk}(k_{t+1},1) z_t^{\rho} e^{\frac{\sigma_z^2}{2}} \left[1 + (\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho \ln z_t + \sigma_z^2)}{\sigma_z}\right)\right] - \right. \\
&\left. - \frac{\mathrm{d}R_{t+1}^f}{\mathrm{d}k_{t+1}} \left[1 + (\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho \ln z_t}{\sigma_z}\right)\right] \right\} - u''(c_t) \Big\}.
\end{aligned}
\tag{1.11}
$$

The divisor is positive while the dividend is negative because expected productivity under the cutoff suggests a loss. The whole fraction is, as a result, negative, showing that the more concerns the agent has on losses relative to gains, the less capital she accumulates in equilibrium. Likewise,

$$
\begin{aligned}
\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\eta} &= \beta E_t \left[v(D_{t+1})\right] \div \Big\{-\beta E_t \Big[u''(c_{t+1}) \left(R_{t+1}^k\right)^2 + u'(c_{t+1}) Z_{t+1} F_{kk}(k_{t+1},1)\Big] - \\
&- u''(c_t) - \eta\beta \left\{ F_{kk}(k_{t+1},1) z_t^{\rho} e^{\frac{\sigma_z^2}{2}} \left[1 + (\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho \ln z_t + \sigma_z^2)}{\sigma_z}\right)\right] - \right. \\
&\left. - \frac{\mathrm{d}R_{t+1}^f}{\mathrm{d}k_{t+1}} \left[1 + (\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho \ln z_t}{\sigma_z}\right)\right] \right\} \Big\}.
\end{aligned}
\tag{1.12}
$$

The quotient has the same sign as the dividend. The agent will reduce investment with a higher weight on the loss aversion utility if she expects a loss conditional on t's productivity. A higher weight means that the loss aversion component gets more important to the total utility, so that an expected loss harms her welfare more.

The following proposition summarizes comparative statics of loss aversion parameters on equilibrium allocations.

Proposition 1. *Fixing the current state, (k_t, z_t), and other parameters,*

(1) as the loss aversion degree, λ, increases, the household reduces investment in risky assets;

(2) as the relative weight of loss aversion utility, η, increases, the household reduces investment in risky assets if she expects a loss and vice versa.

1.2.6 Steady State

This subsection briefly discusses the effect of loss aversion on agents' decisions in the deterministic and risky steady state. It shows that the investment choice in a loss aversion environment is the same as the case without loss aversion in the deterministic steady state. However, in the risky steady state, the household chooses different allocations thanks to loss aversion.

Proposition 2. *In the deterministic steady state of competitive equilibrium, with and without loss aversion, the representative household determines the same consumption and investment allocations.*

Appendix A states the proof to the proposition. The intuition is simple. I formulate loss aversion in an intertemporal environment based on uncertainty. The deterministic steady state rules out unpredictable shocks on future productivity. The household no longer gains or loses because equity premium is constantly zero. The second expectation term regarding loss aversion of (1.9) fails to come into effect.

The deterministic steady state fails to capture how loss aversion affects dynamic properties of competitive equilibrium. Instead, I study the risky steady state defined by Coeurdacier et al. (2011): the risky steady state is the point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is 0 at this date. From Proposition 1, I obtain the following corollary:

Corollary. *An increase in the loss aversion degree, λ, reduces investment in risky assets in the risky steady state equilibrium.*

In the long run, loss aversion causes a reduction in the stock of risky assets compared to the non-loss-aversion scenario. It differs from the effect of risk aversion because stronger precautionary saving motives from higher risk aversion raise the long-run level of assets.

1.3 Inefficiency of Competitive Equilibrium

This section discusses efficiency of competitive equilibrium in my model. I first construct a simplified two-period model to analyze the effect of increasing the capital stock on utility following Davila et al. (2012). It indicates that in this simple model the competitive equilibrium is inefficient when agents are loss averse. I then return to the baseline model and compare the allocations in the competitive equilibrium and the constrained optimum. The result shows that in general, the competitive equilibrium differs from the constrained optimum in its characterization.

1.3.1 Inefficiency of Equilibrium in a Two-Period Model

Consider an economy with a representative household that lives two periods. In the first period, period 0, the household is endowed with e units of output and chooses the amount of consumption, c_0, capital stock, k, and bonds, a, to maximize her utility from consumption and loss aversion utility from expected gains. In the second period, period 1, she obtains utility only from period 1's consumption after receiving labor and asset income. In period 0, aggregate productivity is normalized to 1; in period 1, a productivity shock, σ_z, hits the economy, causing a stochastic productivity Z_1.

The representative firm uses labor and capital, and produces output with a constant-returns-to-scale Cobb-Douglas production technology. As analyzed previously, the household supplies her labor completely, $n = 1$, in equilibrium. The wage and the rental rate equal the marginal products of inputs with labor evaluated at 1. The household holds zero bond in equilibrium, $a = 0$.

The competitive equilibrium in this two-period model is a sequence of prices $\{w_1, r_1, R^f\}$ and a sequence of allocations $\{k, a\}$ such that

(1) given prices, k and a solve

$$\max_{k,a} u(e - k - a) + \eta\beta E_0 \left[v\left(R_1 - R^f\right) k\right] + \beta E_0 \left[u\left(w_1 + R_1 k + R^f a\right)\right];$$

(2) $r_1 = z_1 F_k(k, 1)$ and $w_1 = z_1 F_n(k, 1)$;

(3) $a = 0$.

The social planner chooses an allocation, meaning that they can only adjust the level of capital stock to affect welfare. Denote the total utility across two periods as U. I define constrained efficiency of an allocation as follows:

Definition. *The allocation k is constrained efficient if it is feasible (i.e., $k \in [0, e]$) and if there is no other feasible allocation k' such that $U(k') > U(k)$.*

Whether the competitive equilibrium is constrained efficient depends on whether the planner can improve the welfare by dictating a different level of capital. Thus I consider the effect of increasing capital on the total utility following Davila et al. (2012). Differentiating the total utility, I obtain

$$\begin{aligned}
\mathrm{d}U = & -u'(e-k-a)(\mathrm{d}k+\mathrm{d}a)+ \\
& +\eta\beta\left\{E_t\left[v\left(R_1-R^f\right)\right]\mathrm{d}k+kE_t\left[\frac{\mathrm{d}v\left[\left(R_1-R^f\right)\right]}{\mathrm{d}R_1}+\frac{\mathrm{d}\left[v\left(R_1-R^f\right)\right]}{\mathrm{d}R^f}\right]\right\}+ \\
& +\beta E_t\left[u'\left(w_1+R_1k+R^fa\right)\left(\mathrm{d}w_1+R_1\mathrm{d}k+k\mathrm{d}R_1+R^f\mathrm{d}a+a\mathrm{d}R^f\right)\right]
\end{aligned}$$

The first-order conditions for the household's maximization problem read

$$u'\left(e-k-a\right)=\beta E_t\left[u'(w_1+R_1k+R^fa)R_1\right]+\eta\beta E_t\left[v\left(R_1-R^f\right)\right],$$

$$u'\left(e-k-a\right)=\beta E_t\left[u'(w_1+R_1k+R^fa)\right]R^f.$$

I simplify the expression of $\mathrm{d}U$ by inserting these conditions and then obtain

$$\begin{aligned}
\mathrm{d}U = & \,\eta\beta\left\{E_t\left[v\left(R_1-R^f\right)\right]\mathrm{d}k+kE_t\left[\frac{\mathrm{d}v\left[\left(R_1-R^f\right)\right]}{\mathrm{d}R_1}+\frac{\mathrm{d}\left[v\left(R_1-R^f\right)\right]}{\mathrm{d}R^f}\right]\right\}+ \\
& +\beta E_t\left[u'\left(w_1+R_1k+R^fa\right)\left(\mathrm{d}w_1+k\mathrm{d}R_1+a\mathrm{d}R^f\right)\right].
\end{aligned}$$

Note that $\mathrm{d}R_1=\mathrm{d}r_1=z_1F_{kk}(k,1)\mathrm{d}k$ and $\mathrm{d}w_1=z_1F_{nk}(k,1)\mathrm{d}k$. Since the production technology F is homogeneous of degree 1, $F_n(k,1)+kF_k(k,1)=F(k,1)$. Differentiating both handsides with respect to k, we obtain $F_{nk}(k,1)\mathrm{d}k+kF_{kk}(k,1)\mathrm{d}k=0$. Therefore,

$$\mathrm{d}w_1+k\mathrm{d}R_1=z_1F_{nk}(k,1)\mathrm{d}k+z_1kF_{kk}(k,1)\mathrm{d}k=0.$$

This subsection focuses on the efficiency of equilibrium, thus I talk about the impact of a small deviation from equilibrium. After inserting market equilibrium conditions and above expressions, I differentiate the return to riskfree assets from the first-order condition with respect to capital evaluating $a=0$, and get

$$\frac{\mathrm{d}R^f}{\mathrm{d}k}=\frac{-u''\left(e-k\right)-\beta R^fE_t\left[u''\left(w_1+R_1k\right)R_1\right]}{\beta E_t\left[u'\left(w_1+R_1k\right)\right]}>0, \tag{1.13}$$

$$
\begin{aligned}
&\frac{\mathrm{d}U\mid_{\text{equilibrium}}}{\mathrm{d}k} \\
&= \eta\beta k \left[\int_0^{z^{idf}} \lambda \left(\frac{\mathrm{d}R^k}{\mathrm{d}k} - \frac{\mathrm{d}R^f}{\mathrm{d}k} \right) \mathrm{dF}_{Z_1|Z_0=1}(z) + \lambda \left(z^{idf} F_k(k,1) + 1 - \delta - R^f \right) \frac{\mathrm{d}z^{idf}}{\mathrm{d}k} + \right. \\
&\qquad \left. + \int_{z^{idf}}^{\infty} \left(\frac{\mathrm{d}R^k}{\mathrm{d}k} - \frac{\mathrm{d}R^f}{\mathrm{d}k} \right) \mathrm{dF}_{Z_1|Z_0=1}(z) - \left(z^{idf} F_k(k,1) + 1 - \delta - R^f \right) \frac{\mathrm{d}z^{idf}}{\mathrm{d}k} \right] \\
&= \eta\beta k \left\{ \left[1 + (\lambda - 1)\Phi\left(\frac{\ln z^{idf} - \sigma_z^2}{\sigma_z} \right) \right] e^{\frac{\sigma_z^2}{2}} F_{kk}(k,1) - \left[1 + (\lambda - 1)\Phi\left(\frac{\ln z^{idf}}{\sigma_z} \right) \right] \frac{\mathrm{d}R^f}{\mathrm{d}k} \right\}.
\end{aligned} \tag{1.14}
$$

$\left(z^{idf}F_k(k,1) + 1 - \delta - R^f\right)$ equals 0 after evaluating the expression of cutoff, z^{idf}, which implies that a change in capital does not affect the total utility through the channel of changing the cutoff value. These two terms in the last line are both negative. It manifests that a reduction in capital exerts a positive impact on the welfare in this two-period model.

The above expression uncovers how the variation of capital in equilibrium affects the total utility with loss aversion. First, if the household does not obtain loss aversion utility from expected gains, $\eta = 0$, the competitive equilibrium is optimal, which directly results from the first welfare theorem. Second, as long as the household is loss averse, a decrease in capital from the equilibrium level heightens the welfare since the second-order derivatives of the production function F and the utility from consumption u are negative. The following proposition summarizes the statement.

Proposition 3. *In a two-period model, the competitive equilibrium is suboptimal as long as the household is loss averse; besides, the equilibrium capital stock is higher than the constrained optimal level.*

As the planner reduces the capital stock, the expected return to capital increases. Meanwhile, the household expects period 1's consumption to be lower, or, higher marginal benefit of saving. The return to safe assets must be sufficiently low to clear the bond market. Then the expected equity premium increases, resulting in a higher loss aversion utility conditional on exogenous possibility of productivity.

From another perspective, the household chooses an inefficient allocation because of pecuniary externality. The atomistic household does not internalize the prices and only optimizes her own allocations regardless of the effect of her action on the whole economy through the channel of prices. On the contrary, the social planner considers the price effect and moderates asset holdings to affect asset prices and improve the loss aversion utility.

1.3.2 Characterize Constrained Efficiency

This subsection provides the necessary condition of constrained efficiency, the first-order condition of the household's maximization problem subject to the feasibility constraint. Following the same notation in the last subsection, $\frac{\mathrm{d}U}{\mathrm{d}k} = 0$. I characterize constrained efficiency for the baseline model with infinite periods. The constrained optimum is, in general, different from the competitive equilibrium.

I consider a constrained-efficient social planner who assigns capital stock for the representative agent facing identical preferences. The key issue in constructing the social planner's problem is how to determine the asset prices because they appear in the utility function. Factor prices and interest rate are determined by market equilibrium conditions in line with Bianchi and Mendoza (2013). The constrained optimum is the solution to

$$\max_{c_t, k_{t+1}} \quad u(c_t) + \eta\beta k_{t+1} E_t \left[v(D_{t+1}) \right]$$

subject to

$$c_t + k_{t+1} = Z_t F(k_t, 1) + (1 - \delta) k_t,$$

where

$$R^f_{t+1} = \frac{u'(c_t)}{\beta E_t \left[u'(c_{t+1}) \right]} \text{ and } D_{t+1} = R^k_{t+1} - R^f_{t+1}$$

.

The first-order condition for the social planner's problem is

$$\begin{aligned}
u'(c_t) = {} & \beta E_t\left[R^k_{t+1}u'(c_{t+1})\right] + \eta\beta E_t\left[v(D_{t+1})\right] + \\
& + \eta\beta k_{t+1}\left\{\left[1+(\lambda-1)\Phi\left(\frac{\ln z^{idf}_{t+1}-(\rho\ln z_t+\sigma_z^2)}{\sigma_z}\right)\right] z_t^{\rho} e^{\frac{\sigma_z^2}{2}} F_{kk}(k_{t+1},1) - \right. \\
& - \left.\left[1+(\lambda-1)\Phi\left(\frac{\ln z^{idf}_{t+1}-\rho\ln z_t}{\sigma_z}\right)\right]\frac{\mathrm{d}R^f_{t+1}}{\mathrm{d}k_{t+1}}\right\} - \\
& - \eta\beta E_t\left\{k_{t+2}\left[1+(\lambda-1)\Phi\left(\frac{\ln z^{idf}_{t+2}-\rho\ln z_{t+1}}{\sigma_z}\right)\right]\frac{\mathrm{d}R^f_{t+2}}{\mathrm{d}k_{t+1}}\right\} - \\
& - \eta k_t\left[1+(\lambda-1)\Phi\left(\frac{\ln z^{idf}_{t}-\rho\ln z_{t-1}}{\sigma_z}\right)\right]\frac{\mathrm{d}R^f_{t}}{\mathrm{d}k_{t+1}},
\end{aligned} \tag{1.15}$$

where

$$\frac{\mathrm{d}R^f_{t+1}}{\mathrm{d}k_{t+1}} = \frac{-u''(c_t)-\beta R^f_{t+1}E_t\left[u''(c_{t+1})R^k_{t+1}\right]}{\beta E_t\left[u'(c_{t+1})\right]} > 0,$$

$$\frac{\mathrm{d}R^f_{t+2}}{\mathrm{d}k_{t+1}} = \frac{u''\left(c_{t+1}\right)R^k_{t+1}}{E_{t+1}\left[u'\left(c_{t+2}\right)\right]} < 0,$$

$$\frac{\mathrm{d}R^f_{t}}{\mathrm{d}k_{t+1}} = \frac{E_{t-1}\left[u''\left(c_t\right)\right]R^f_t}{E_{t-1}\left[u'\left(c_t\right)\right]} < 0.$$

The first line of (1.15) duplicates the equilibrium condition, yet there are more terms in the optimum characterization. Line 2 and 3 suggest the same effect of a deviation from a certain capital level as in the two-period model. Line 4 shows that a different level of capital, k_{t+1}, affects the loss aversion utility in period $t+1$ since capital then will evolve through a different path, resulting in a different capital level decided in $t+1$, k_{t+2}, which further leads to different returns to assets. The last line indicates that a change in the allocation in the future even influences the welfare in the past in a perfect foresight model. In addition, increasing capital, k_{t+1}, raises the loss aversion utility in $t+1$ and in $t-1$ since returns to safe assets decline accordingly. The positive influence of higher capital exerted in $t+1$ and $t-1$ makes the optimization problem more complicated to analyze than in a two-period model. I turn to numerical analysis in next section. However, the constrained optimum is, in general, different from the competitive equilibrium, unless all last four lines in the characterization of constrained optimum counterbalance each other every period. Section 4 shows that even the risky steady state of equilibrium differs from that of optimum.

1.4 Numerical Analysis

This section presents numerical results on comparative statics of loss aversion parameters on equilibrium and a comparison between the competitive equilibrium and the constrained optimum. I start from describing how I calibrate the model. The calibrated model features lower capital stock in the risky steady state of equilibrium as loss aversion parameters increase. The equilibrium differs from the constrained optimum within the range of parameter values I attempt.

1.4.1 Calibration

I assume the consumption preference as a standard CRRA function, $u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}$, where θ determines the degree of risk aversion and $\theta > 0$. The production function is a Cobb-Douglas production function, $Y_t = Z_t k_t^{\alpha} n_t^{1-\alpha}$, where α denotes the capital share of output and $\alpha \in (0, 1)$.

Most of the parameter values that I use are in line with yearly data of the United States or other estimates in the literature. I set capital share of income $\alpha = 0.36$. My selection of δ is 0.1, which is in accordance to the annual depreciation rate.

Furthermore, the discount factor, β, is calibrated to be 0.97 so that in a non-loss-aversion economy, given the above parameters, the ratio of capital over output is roughly 2.7 in the deterministic steady state. I keep the discount factor unchanged in this section when I introduce loss aversion in the model since loss aversion plays no role in the deterministic steady state.

The risk aversion degree, θ, is set to be 1, which is common in the macroeconomic literature. Note that equity premium generated from the US data is 6 percentage points which only unrealistically high risk aversion degrees could match the moment in the business cycle framework. Although my equilibrium simulation cannot generate the realistic equity premium, it clearly shows that equity premium rises to a higher magnitude. It also manifests that even small equity premium is able to considerably affect the individual behavior and the whole economy. From another point of view, if I add other elements to the model or simply calibrate the current model to produce the reasonable equity premium, we will see a much larger effect.

I assume that the autoregressive parameter for the technology shock, ρ_z, and the standard deviation for innovations, σ_z, are 0.81 and 0.04, respectively, in keeping with the real business cycle literature.

Experiments report that the relative weight, η, is nearly 1 and some applications in the literature also take 1 as a reasonable parameter value or as the upper bound of a series of values. Research documents that the loss aversion degree, λ, varies between 2 and 3, most of which are estimated approximately equal to 2.5. So I set baseline values of two parameters as 1 and 2.5. Table 2.1 summarizes the parameter values for the baseline model.

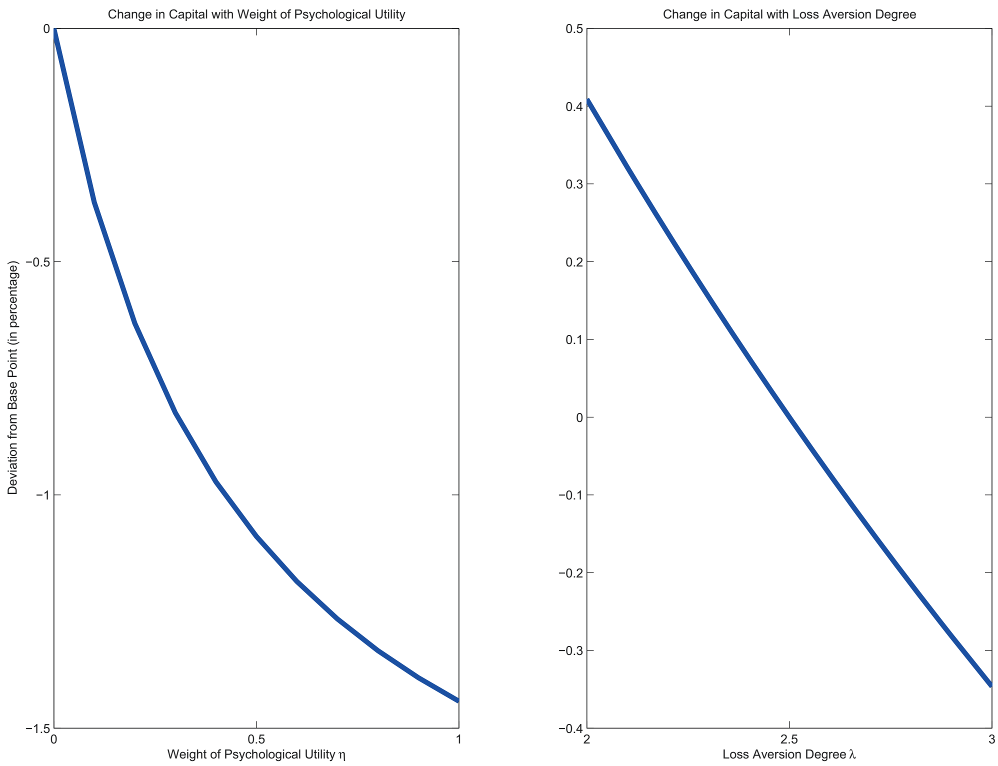
Table 1.1: Parameter Values for Baseline Model

Parameters	**Values**	**Descriptions**
α	0.36	capital share of output
β	0.97	discounted rate
δ	0.1	depreciation rate
θ	1	risk aversion degree
ρ_z	0.81	autocorrelation of productivity
σ_z	0.04	standard deviation of productivity shock
λ	2.5	baseline loss aversion degree
η	1	baseline relative weight of loss aversion utility over consumption

1.4.2 Comparative Statics in Risky Steady State

My first exercise aims to understand the effect of loss aversion on equilibrium prices and allocations with aggregate shocks, so I study comparative statics of loss aversion parameters by computing the risky steady state of prices and allocations in equilibrium. I first stabilize the loss aversion degree, λ, equal to 2.5 and change the relative weight, η, from 0 to 1 with an interval of 0.1 to see how more concerns on investment fluctuations impacts the behavior. I then fix η as 1 and change λ from 2 to 3 with an interval of 0.1 to see the influence of greater loss aversion.

Figure 1.1 presents the change in the risky steady state of capital as the two loss aversion parameters vary. The values reported in the left panel are percentage changes from the risky steady state of a non-loss-aversion model while the right panel depicts the deviation from the base point set to be the risk steady state in the baseline model. Supposing that positive and negative shocks hit the economy symmetrically, loss aversion provokes a negative summed utility of accumulating capital over the whole business cycle given a positive relative weight. An increase in either of these two parameter values



All the values are percentage changes. In the left panel the base is set to be the risky steady state in a standard model; in the right panel the base is the risky steady state in the baseline model with $\eta = 1$ and $\lambda = 2.5$.

Figure 1.1: Comparative Statics for Risky Steady State

enlarges the gap between the disutility from losses and the utility from gains, which discourages the household from investing in capital.

Table 1.2: Risky Steady State in Equilibrium (in percentage)

Variables	**Baseline** ($\eta = 1,\ \lambda = 2.5$)	**Higher Loss Aversion** ($\eta = 1,\ \lambda = 3$)
Capital	-1.44	-1.78
Output	-0.52	-0.65
Consumption	-0.18	-0.22
Gross Return to Capital	0.12	0.15
Gross Return to Bonds	-0.00	-0.00
Equity Premium	0.11	0.14

All the values are percentage changes. The base is set to be the risky steady state in a standard model.

Table 1.2 exhibits comparative statics of more price and allocation variables. I pick up the statistics from a standard model without loss aversion, from the baseline model, and from a model with higher loss aversion degree, $\lambda = 3$, and report the percentage deviations in models with loss aversion taking as the base the risky steady state in a standard model.

Both a greater relative weight and a greater loss aversion degree lead to lower investment in productive assets, capital, which further induces lower output and consumption in the long run. Capital has a higher return because it becomes scarcer. The upward change in equity premium originates from higher returns to capital and no significant change in returns to bonds.

1.4.3 Comparison of Equilibrium and Constrained Optimum

I solve the competitive equilibrium and the constrained optimum, respectively, given the baseline parameter values. Table 1.3 reports the comparison of selected prices and allocations in risky steady state. It also shows the welfare loss from the constrained optimum to the equilibrium measured by consumption equivalent variation. I construct consumption equivalent, ce, such that in risky steady state,

$$\frac{ce^{1-\theta}}{1-\theta} = \frac{c^{1-\theta}}{1-\theta} + \eta\beta E\left[v(D)\right].$$

Thus, consumption equivalent reflects the total welfare in the long run. I set the base as the risky steady state value of consumption equivalent at optimum.

Table 1.3: Comparison of Equilibrium and Optimum with Loss Aversion

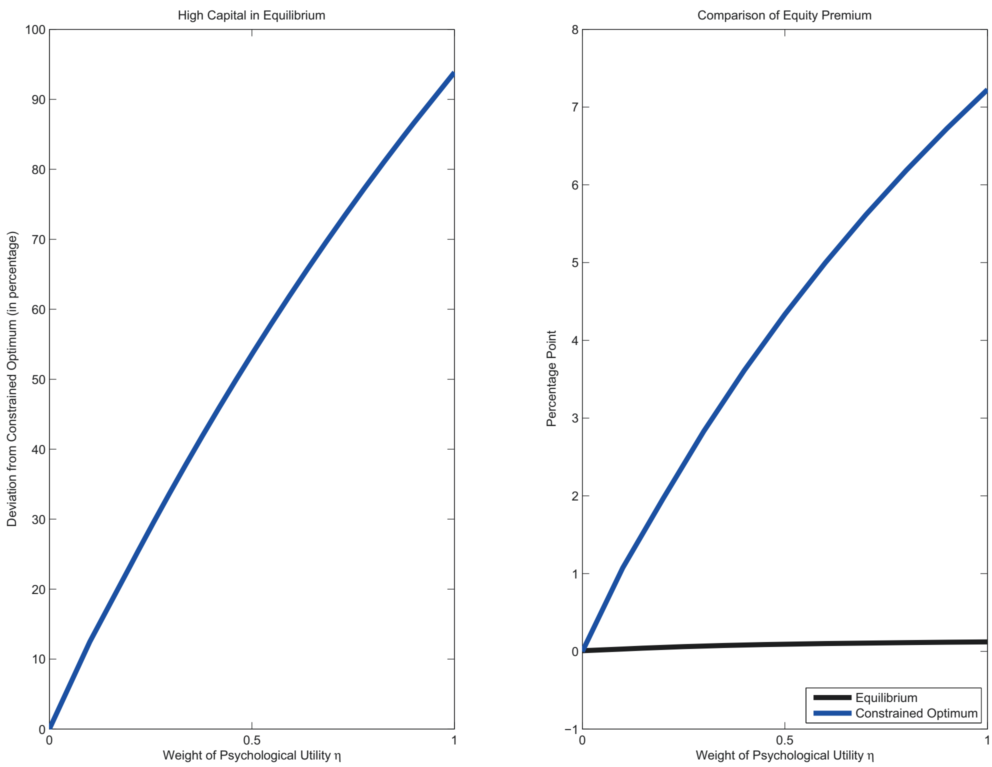
Variables	**Equilibrium**	**Constrained Optimum**
Capital	4.65	2.40
Output	1.74	1.37
Consumption	1.27	1.13
Gross Return to Capital	1.034	1.106
Gross Return to Bonds	1.033	1.033
Equity Premium	0.001	0.07
Welfare Change (Consumption Equivalent Variation)		
	-5.27	

Values of prices and allocations are in levels. The welfare is measured in percentage points. The base is set to be the risky steady state at optimum.

As analyzed before, capital stock in equilibrium with loss aversion, although lower than its counterpart in a standard model, is still higher than the constrained optimum. The household experiences a loss from lower consumption at optimum, yet obtains sufficient compensation from the loss aversion utility because equity premium rises much. Therefore, she is better off when moving from the competitive equilibrium to the constrained optimum.

Obviously, the social optimum equals the competitive equilibrium without any risks even if the model incorporates loss aversion. Thus the numerical result from the baseline model also manifests that aggregate productivity shocks, or fluctuations over the business cycle, incur much more welfare loss than standard models generate. Lucas (1987) finds a negligible welfare gain if removing all the risks. Instead, the welfare gain is large in my model because together with smooth consumption, the household no longer obtains disutility from potential losses from investment.

Figure 1.2 illustrates that the competitive equilibrium in my model generally differs from the constrained optimum. This exercise varies the relative weight of loss aversion utility from 0 to 1 with an interval of 0.1 and calculates the risky steady state of equilibrium and constrained optimum, respectively. The left panel takes the optimum as the base and obtains the percentage change of capital stock in equilibrium. It indicates that the household accumulates capital more than social optimality requires. The capital stock in equilibrium with baseline calibration almost doubles the efficient amount. Consequently,



The left panel plots the percentage change of capital stock in the risky steady state of competitive equilibrium compared to the constrained optimum. The right panel plots equity premium in percentage points in equilibrium and at the optimum.

Figure 1.2: Comparison of Equilibrium and Constrained Optimum

the risk steady state of equity premium in equilibrium is much lower than the counterpart at optimum as the right panel shows.

1.5 Government Intervention

Inefficiency of competitive equilibrium due to loss aversion poses the questions of whether the government should intervene and how. This section answers these questions by adding a government sector to the baseline model and exploring a Ramsey problem. I describe the government's problem directly with functional forms used in Section 4 to uncover Ramsey optimal allocations over the business cycle by quantitative analysis.

1.5.1 Economy

With the government, the feasibility condition of this economy becomes

$$c_t + k_{t+1} + g_t = Y_t + (1-\delta)k_t, \tag{1.16}$$

where $\{g_t\}_{t=0}^{\infty}$ represents a sequence of government purchases.

Government consumption is modelled as an exogenous AR(1) process,

$$\ln g_{t+1} = (1-\rho_g)\ln \bar{g} + \rho_g \ln g_t + \sigma_g \epsilon_{t+1}^g, \tag{1.17}$$

where ρ_g denotes the autoregressive parameter for government consumption evolution, σ_g represents the standard deviation of one-time innovation on government spending and ϵ_{t+1}^g is distributed as an independent standard normal, $\epsilon_{t+1}^g \sim N(0,1)$, for any $t \geq 0$. $\bar{g}$ captures the long-run level of government consumption.

1.5.2 Government

The government finances its expenditure by levying distortionary taxes on capital income at rate τ_t^k and a lump-sum tax T_t. I assume that the tax rate, τ_t^k, is predetermined according to the information updated until period $t-1$, which implies that tax policy is non-state-contingent. This assumption reflects the norm of fiscal policy: policymakers usually propose and decide a taxation policy before the policy enters into force. The non-state-contingent tax rate makes it possible to derive a simple form of z_{t+1}^{idf}. The lump-sum tax is state-contingent to support the Ramsey optimal allocation. I further assume that the government does not hold any debt since the lump-sum tax is available. The government budget constraint is

$$T_t + \tau_t^k r_t k_t = g_t. \tag{1.18}$$

In a standard model, the government naturally applies the nondistortionary lump-sum tax only to finance its spending. I will show that the government in my model, on the contrary, has incentive to distort asset prices through capital income taxes to correct the inefficiency.

1.5.3 Household

At period t, the household receives income from labor supply, capital rental and interest of private bonds, learns the news of next period's taxation proposal and then determines the amount of consumption, labor supply, capital accumulation and the purchase of next period's bonds. The representative household compares the expected gross returns to two assets in the fashion described in previous sections except that the household considers the gross return to risky assets net of capital taxes, $R_t^k = (1-\tau_t^k)r_t + 1 - \delta$. The indifferent productivity level to invest in risky and riskfree assets,

$$z_{t+1}^{idf} = \frac{R_{t+1}^f - 1 + \delta}{(1-\tau_{t+1}^k)\alpha k_{t+1}^{\alpha-1}}.$$

Providing that the government imposes a high tax on capital income, the value of z_{t+1}^{idf} will be large. Hence, a higher capital tax rate not only undermines the desire of accumulating capital, but also directly raises the expectation of losses.

With government, the household budget constraint becomes

$$c_t + k_{t+1} + a_{t+1} = w_t n_t + \left[(1-\tau_t^k)r_t + 1 - \delta\right] k_t + R_t^f a_t - T_t. \tag{1.19}$$

1.5.4 Competitive Equilibrium Conditions

The determination of factor prices remains unchanged as (1.7) and (1.8). Euler equations incorporate identical terms as (1.9) and (1.10) with the exception that the expected loss aversion utility per unit of capital, $E_t\left[v(D_{t+1})\right]$, given the tax rate, is rewritten as,

$$\begin{aligned} E_t\left[v(D_{t+1})\right] = &\left(1-\delta-R_{t+1}^f\right)\left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho \ln z_t}{\sigma_z}\right)\right] + \\ &+ (1-\tau_{t+1}^k)\alpha k_{t+1}^{\alpha-1} z_t^{\rho} e^{\frac{\sigma_z^2}{2}}\left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho\ln z_t + \sigma_z^2)}{\sigma_z}\right)\right]. \end{aligned} \tag{1.20}$$

1.5.5 Ramsey Problem

This subsection builds up the Ramsey problem, presents quantitative dynamics of the optimal policy and discusses the underlying mechanism.

Formulation of Ramsey Problem

I first clarify the timing of fiscal policy in this Ramsey problem. At time 0, the government sets fiscal policy of every period from time 1 onwards. Particularly, the capital income tax rate of $t+1$ depends on the state of t, which is different from the normal setting in the Ramsey literature. For example, the usual formulation requires that the government should announce at time 0 to levy a high tax when the present productivity is high; the government in my model, instead, claims to tax much when the productivity of last period appears high, even if the current productivity ends up at a low level. I formulate the Ramsey problem as follows:

$$\max_{\tau_{t+1}^k, T_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\theta}}{1-\theta} + \eta\beta k_{t+1} E_t \left[v(D_{t+1}) \right] \right\}$$

subject to

$$\begin{aligned} T_t + \tau_t^k r_t k_t &= g_t, \\ c_t + k_{t+1} + g_t &= z_t k_t^{\alpha} + (1-\delta) k_t, \\ c_t^{-\theta} &= \beta E_t \left[R_{t+1}^k c_{t+1}^{-\theta} \right] + \eta\beta E_t \left[v(D_{t+1}) \right], \\ c_t^{-\theta} &= \beta R_{t+1}^f E_t \left[c_{t+1}^{-\theta} \right]. \end{aligned}$$

Parameter values

$\bar{g}$ is chosen to be 0.31 so that the government spending accounts for roughly 18% of output in the steady state of Ramsey optimal allocations in line with the US observation. I set $\rho_g = 0.89$ and $\sigma_g = 0.07$ as Chari et al. (1994) do. I summarize the parameters for the government sector in Table 1.4.

Table 1.4: Parameter Values for Government Sector

Parameters	**Values**	**Descriptions**
$\bar{g}$	0.31	long-run government spending
ρ_g	0.89	persistence of government consumption
σ_g	0.07	standard deviation of innovation on government spending

Results on Optimal Policy

Table 1.5 reports business cycle statistics of optimal policy instruments and other variables. All mean values are expressed in levels except that the capital income tax rate is measured in percentage points. The relative standard deviations are computed by dividing standard deviations in levels by their respective means. As a comparison, Table 1.5 also records statistics of the same variables in a standard model.

Table 1.5: Statistics under Optimal Policy

Variables	**Baseline**		**Non-Loss-Aversion**	
	mean	**rsd.**	**mean**	**rsd.**
Capital Tax Rate	60.76	0.27	0	0
Lump-sum Tax	0.20	0.72	0.31	0.15
Capital	1.06	0.11	4.77	0.10
Output	1.02	0.09	1.76	0.09
Consumption	0.60	0.18	0.96	0.10
Gross Return to Capital	1.25	0.02	1.03	0.01
Gross Return to Bonds	1.03	0.05	1.03	0.01

rsd. represents the relative standard deviation. All mean values are expressed in levels expect that capital income tax rate is measured in percentage points.

Table 1.5 presents my major result on optimal policy: in an environment with loss aversion, the government should tax capital income. Zero capital tax rate is suboptimal. To overcome the inefficiency from loss aversion, the government reduces capital stock to reach the optimum, thus it should tax capital considerably even without any idiosyncratic risks or frictions.

A problem of asymmetric taxation comes from the tax exemption of riskfree bonds. I can instead assume that the government taxes the revenue of bonds as well. Yet even with symmetric taxation, the asset tax still discourages the household from investing in risky assets, causing a higher return to capital. In addition, the tax revenue of the government does not increase because of zero bond holdings. Hence a benevolent government would choose asymmetric taxation as a better mechanism design because it enlarges the gap between asset returns and heightens the welfare efficiently.

In the non-loss-aversion model, the government equalizes the lump-sum tax and its spending. In contrast, with loss aversion the government collects tax revenues also from capital income so that it imposes a lighter lump-sum tax.

As for the relative standard deviations, the capital income tax rate in the standard model keeps constant, equal to 0, over the whole cycle. In the baseline model, the government applies it as an effective instrument to fight against the negative impact of fluctuations so that it varies a lot. The lump-sum tax gets more volatile because it no longer pegs the government expenditure, but associated with the productivity shock to balance the budget.

The result matches the argument of constrained efficient allocations. The government should apply its instruments, which are capital income taxes and lump-sum taxes in my model, to reduce the capital from an inefficient equilibrium level to a lower optimal level. The planner faces a tradeoff between lower level of consumption and higher loss aversion utility. The latter dominates the former in this model.

1.6 Conclusion

This paper discusses the behavior of a production economy over the business cycle considering loss aversion, a core concept of prospect theory commonly accepted in behavioral economics. As far as I know, research has never applied prospect theory to a dynamic stochastic general equilibrium framework and policy analysis. My paper fills the gap by modelling loss aversion as another component in addition to consumption in the representative household's preferences. The household expects an extra gain from investing in risky assets, capital, relative to the same amount of riskfree assets, bonds, and acquires positive utility. If she predicts a loss, she gets disutility whose absolute value is greater than that of utility from the same amount of gain. Thus, fluctuations over the business cycle affects the welfare not only indirectly by making consumption volatile, but also directly by altering expectation on asset returns. The latter is generally negative due to asymmetric influences on the utility of gains and losses.

I first focus on the risky steady state equilibrium and show that the more loss aversion and the more concerns on the loss aversion utility, the less investment in risky assets in the risky steady state. I show analytically that the competitive equilibrium is inefficient by considering a constrained optimality problem in a two-period model. The numerical analysis compares the competitive equilibrium and the constrained optimum and confirms the above statement in an infinite-horizon model. This is because an atomic household takes prices as given and fails to think about the impact of her actions on prices. The pecuniary externality creates an inefficient equilibrium since prices enter into preferences of a loss averse household. The equilibrium level of capital is much higher than the optimal level because the household can obtain a higher loss aversion utility from higher equity premium if investing less in capital.

I then add the government sector and investigate the Ramsey optimal policy. Zero capital tax in the long run becomes suboptimal. The government should tax capital accumulation to approach the constrained optimum.

I also find that the welfare loss from fluctuations over the business cycle is much higher than most of estimations in standard models. This is because the household directly obtains disutility from potential losses from investment in addition to more volatile consumption.

Appendix

1.A Proof of Proposition 2

Proof. In deterministic steady state, an arbitrary variable is constant, $x_t = \bar{x}$, and the standard deviation of the shock $\sigma_z = 0$. To prove the proposition, I only need to show that $v(\bar{D}) = 0$. Then the rest follows the same argument in the non-loss-aversion case.

First, from the steady state version of (12), $\bar{R}^f = \frac{1}{\beta}$.

Second, since $v(D_{t+1})$ is a linear function with a kink, we have that at the deterministic steady state, $E_t\left[v(D_{t+1})\right] = \nu \cdot D_{t+1}$, where I denote ν as the slope at steady state. Notice that the slope can be any number between 1 and λ. Then (11) becomes

$$u'(\bar{c}) = \beta \bar{R}^k u'(\bar{c}) + \eta\beta\nu(\bar{R}^k - \bar{R}^f).$$

A simple algebra gives us

$$\beta(u'(\bar{c}) + \eta\nu)\bar{R}^k = u'(\bar{c}) + \eta\beta\nu\bar{R}^f.$$

Recall that $\bar{R}^f = \frac{1}{\beta}$. Thus, $\bar{R}^k = \frac{1}{\beta}$ and $v(\bar{D}) = 0$. $\square$

1.B Simplification of $E_t\left[v(D_{t+1})\right]$

Since $\ln Z_{t+1} = \rho \ln Z_t + \sigma_z \epsilon^z_{t+1}$ and ϵ^z_{t+1} is distributed as a standard normal, Z_{t+1} follows a log-normal distribution conditional on t's information.We replicate here the expression of $E_t\left[v(D_{t+1})\right]$,

$$E_t v(D_{t+1}) = \int_0^{z^{idf}_{t+1}} \lambda(R^k_{t+1} - R^f_{t+1})\mathrm{dF}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) + \int_{z^{idf}_{t+1}}^{\infty} (R^k_{t+1} - R^f_{t+1})\mathrm{dF}_{Z_{t+1}|Z_t=z_t}(z_{t+1}),$$

where the conditional cumulative distribution function of shock Z_{t+1} with the known history until period t, $\mathrm{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) = \Phi\left(\frac{\ln z_{t+1} - \rho \ln z_t}{\sigma_z}\right)$ when $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. We can also derive the conditional probability density function as $\mathrm{f}_{\mathrm{Z_{t+1}|Z_t=z_t}}(z_{t+1}) = \frac{1}{\sigma_z z_{t+1}}\varphi\left(\frac{\ln z_{t+1} - \rho \ln z_t}{\sigma_z}\right)$ with $\varphi(\cdot)$ representing the probability density function of the standard normal distribution.

Let us focus on the first term of $E_t\left[v(D_{t+1})\right]$.

$$\begin{aligned}
&\int_0^{z^{idf}_{t+1}} \lambda(R^k_{t+1} - R^f_{t+1}) d\mathrm{F}_{\mathrm{Z_{t+1}|Z_t=z_t}}(z_{t+1}) \\
&= \lambda \int_0^{z^{idf}_{t+1}} (z_{t+1}F_k(k_{t+1},1) + 1 - \delta - R^f_{t+1})\mathrm{dF}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) \\
&= \lambda[\int_0^{z^{idf}_{t+1}} (1 - \delta - R^f_{t+1})\mathrm{dF}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) + \int_0^{z^{idf}_{t+1}} F_k(k_{t+1},1)\cdot z_{t+1}\mathrm{dF}_{Z_{t+1}|Z_t=z_t}(z_{t+1})] \\
&= \lambda[(1 - \delta - R^f_{t+1})\Phi\left(\frac{\ln z^{idf}_{t+1} - \rho \ln z_t}{\sigma_z}\right) + F_k(k_{t+1},1)\int_0^{z^{idf}_{t+1}} \frac{z_{t+1}}{\sigma_z z_{t+1}}\varphi\left(\frac{\ln z_{t+1} - \rho \ln z_t}{\sigma_z}\right)\mathrm{d}z_{t+1}].
\end{aligned}$$

$$\begin{aligned}
\int_0^{z^{idf}_{t+1}} \frac{z_{t+1}}{\sigma_z z_{t+1}}\varphi\left(\frac{\ln z_{t+1} - \rho \ln z_t}{\sigma_z}\right)\mathrm{d}z_{t+1} &= \int_0^{z^{idf}_{t+1}} \frac{1}{\sqrt{2\pi}\sigma_\epsilon} e^{-\frac{(\ln z_{t+1} - \rho \ln z_t)^2}{2\sigma_z^2}}\mathrm{d}z_{t+1} \\
&= \int_{-\infty}^{\ln z^{idf}_{t+1}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(y_{t+1} - \rho \ln z_t)^2}{2\sigma_z^2}} e^{y_{t+1}} dy_{t+1} (\text{Let } \ln z_{t+1} = y_{t+1}) \\
&= \int_{-\infty}^{\ln z^{idf}_{t+1}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(y_{t+1} - (\rho \ln z_t + \sigma_z^2))^2}{2\sigma_z^2} + \rho \ln z_t + \frac{\sigma_z^2}{2}}\mathrm{d}y_{t+1} \\
&= z_t^\rho e^{\frac{\sigma_z^2}{2}} \int_{-\infty}^{\ln z^{idf}_{t+1}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(y_{t+1} - (\rho \ln z_t + \sigma_z^2))^2}{2\sigma_z^2}}\mathrm{d}y_{t+1} \\
&= z_t^\rho e^{\frac{\sigma_z^2}{2}} \Phi\left(\frac{\ln z^{idf}_{t+1} - (\rho \ln z_t + \sigma_z^2)}{\sigma_z}\right).
\end{aligned}$$

Thus,

$$\int_0^{z_{t+1}^{idf}} \lambda(R_{t+1}^k - R_{t+1}^f) \mathrm{dF}_{Z_{t+1}|Z_t=z_t}(z_{t+1})$$
$$= \lambda\left[(1-\delta-R_{t+1}^f)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho\ln z_t}{\sigma_z}\right) + F_k(k_{t+1},1)z_t^{\rho}e^{\frac{\sigma_z^2}{2}}\Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho\ln z_t + \sigma_z^2)}{\sigma_z}\right)\right].$$

With the same argument, we calculate the second term as well. Summing up two parts gives us the result:

$$\begin{aligned} E_t\left[v(D_{t+1})\right] =& F_k(k_{t+1},1)z_t^{\rho}e^{\frac{\sigma_z^2}{2}}\left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho\ln z_t + \sigma_z^2)}{\sigma_z}\right)\right] + \\ &+ (1-\delta-R_{t+1}^f)\left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho\ln z_t}{\sigma_z}\right)\right]. \end{aligned}$$

Chapter 2

When to Tax Capital: Fiscal Policy with Idiosyncratic Investment Risks and Heterogeneous Agents

2.1 Introduction

Empirical studies find that wealth is highly concentrated among the rich. Heathcote et al. (2010) document that the richest 10 percent of households in the US hold 59 percent of aggregate net worth in 2007. The net worth tends to be invested in an undiversified portfolio, even for the rich who in general have more investment options. Carroll (2001) find that portfolios of the rich are heavily skewed toward risky assets, especially investment in their own private-held businesses. Poor diversification in investment implies high idiosyncratic investment risks, which affect saving and investment choices, the aggregate economy and the welfare. Furthermore, the idiosyncratic investment risks increase in the recession, causing a larger difference in the performance of returns to investment as shown by Bloom et al. (2016). It may amplify the drop in aggregate investment and output. Thus idiosyncratic investment risks may provide implications on adjusting the capital income tax or other policy instruments related to investment over the business cycle.

This paper examines the effect of fiscal policy on investment and on the welfare of heterogeneous agents over the business cycle with a model featuring the aforementioned facts about wealth and investment. Specifically, I ask when to increase the capital income tax rate. Due to idiosyncratic investment risks, an increase in the capital income tax rate may exert a larger impact on the investment and on the welfare of investors than in representative-agent models. The change in the tax policy further influences output, the wage rate and the welfare of agents who receive labor earnings as the main income source.

I develop a model that incorporates heterogeneous agents, business cycle and a government sector which decides fiscal policy in the framework of Angeletos (2007) considering idiosyncratic investment risks. My model features two types of risk-averse agents: entrepreneurs and hand-to-mouth workers. I allow entrepreneurs to invest in their own firms, or alternatively in non-state-contingent government bonds, but not in the private firms held by other entrepreneurs. Private firms suffer from aggregate productivity shocks and independent and identically distributed idiosyncratic investment risks whose variance rises in the recession, so that low total factor productivity enlarges the gap in business income. Together with i.i.d idiosyncratic investment risks, the aggregate shocks exert an impact on the decision of how much to consume and to save (saving choice) and of how much investment on risky but productive assets and on riskfree assets (portfolio choice). Each period all entrepreneurs make identical choices because of i.i.d idiosyncratic investment risks and no trade in capital among entrepreneurs. Hand-to-mouth workers supply labor inelastically to the labor market. Their income differs due to idiosyncratic labor income risks. A government levies labor and capital taxes and issues government bonds. Specifically, the government sets rules of capital tax and bonds as functions of log-deviation of output fixing the steady state level and lets the labor tax rate balance the budget. These policy instruments influence entrepreneurs more directly since they can save.

I show that to some extent the model matches the level and cyclical behavior of the US income distribution. The simulated income distribution features large income shares in top income groups and small income shares in bottom income groups as shown in the Current Population Survey data. As for the cyclical property of income distribution, the simulated result qualitatively matches most of the correlations of income shares with output and 95/50 and 50/20 ratios.

I apply the model to carry out a policy experiment in which the government obeys specific policy rules and chooses the parameter values indicating responses of capital tax rate and debt to output. I first calibrate these parameters to the US fiscal policy data as the baseline. Then I evaluate different combinations of counterfactual fiscal policy instruments. I find that in my heterogeneous-agent model the choice of capital tax policy or debt policy creates welfare conflicts between entrepreneurs and workers. A policy which specifies a high capital tax rate and a low debt level in the recession benefits entrepreneurs while it harms workers' welfare. The mean level of bonds rises while the average capital tax rate declines compared to the baseline. More government bonds give entrepreneurs insurance and crowd out capital, yet less tax burden encourages entrepreneurs to invest more in capital. The policy slightly raises the average capital stock and the average wage rate. The mean consumption of entrepreneurs increases due to less tax burden and higher asset income. The government budget balance requires a higher average labor tax rate, which causes a decrease in the mean consumption of workers. Meanwhile, the volatility of consumption of entrepreneurs greatly increases under the policy combination. Yet for entrepreneurs, the increase in the mean consumption outweighs the welfare loss from a higher volatility. Workers see less mean consumption and a higher volatility. Thus, entrepreneurs are better off and workers are worse off.

I ask when the government should tax capital more and issue more debt, and in particular, whether to do it during a boom or a bust. In the classical business cycle framework, the seminal paper by Chari et al. (1994) shows that the correlation of an optimal capital income tax rate with technology shock is negative with uncontingent debt in their baseline model. In addition, when there is a negative innovation to the technology shock or a positive innovation to government consumption, there is a positive innovation in the tax on private assets. My result on fiscal policy seems to confirm their finding: The policy that maximizes the utilitarian social utility features an increase by 0.45 percentage point in the capital tax rate and an increase by 0.37 percent in the debt level as the output drops by one percent. Nonetheless, the welfare conflict indicates that workers prefer a capital tax rate which increases less in the recession. The socially preferred policy may reduce the capital tax rate in the bust if the government, instead of adopting the utilitarian welfare criterion, puts more weight on workers. Therefore, it is possible to overturn the result of Chari et al. (1994) such that the government should cut the capital income tax rate in the recession.

I study the impulse responses of aggregate variables to a standard deviation of negative aggregate productivity shock under the baseline policy combination targeted to the data and the other three policy combinations optimizing entrepreneurs, workers and all agents, respectively. These policy combinations affect the behaviors of aggregate variables after the shock. In general, the higher capital tax rate and the lower debt when the adverse shock hits, the higher capital and output in the early stage after the shock. However, the recovery of capital slows down and falls behind the counterparts under other policy combinations because the average return to capital is lower. This policy also reduces the consumption of entrepreneurs most in the beginning and cause the slowest recovery. Under the high capital tax and the low debt level, the labor tax rate rises, which lowers the consumption of workers more than other policies. Yet since the output and the wage recover faster, so does the consumption of workers.

To see if my results are robust to the choice of the policy instrument that balances the government budget, I keep the labor tax rate constant and study the cyclical property of debt on welfare with the consumption tax rate only levied on entrepreneurs to balance the government budget. I find that the welfare conflict still occurs in this experiment when the government chooses debt policy. Entrepreneurs prefer a low debt level in the recession while workers prefer a slightly high level. Workers, like entrepreneurs, favor a procyclical capital tax policy, although the capital tax rate favored by workers rises in the recession less than the one preferred by entrepreneurs.

I examine whether the result of the welfare conflict is robust to constant volatility of idiosyncratic investment risks and no idiosyncratic investment risks. I find that the welfare conflict qualitatively disappears in this two exercises.

My paper contributes to the literature of uninsured idiosyncratic investment risks, which has demonstrated theoretically that idiosyncratic investment risks influence the steady state of the aggregate economy or on the stationary distribution of income, earnings or wealth. These studies find that market incompleteness may lead to underaccumulation or overaccumulation of capital, depending on the parameterization (Angeletos (2007); Angeletos and Calvet (2006); Covas (2006); Meh and Quadrini (2006)). Other scholars apply the framework with idiosyncratic investment risks to explain questions about fiscal policy (Angeletos and Panousi (2009), Panousi (2010)), international difference of growth (Angeletos and Panousi (2011)) and the like. Since aggregate shocks are missing, their studies are silent about the cyclical behavior or the welfare change over the business cycle. My paper, on the contrary, adds aggregate shocks to the framework of Angeletos (2007). The classic Krusell and Smith (1998) framework requires the entire wealth distribution as a state variable. Since the wealth distribution is an infinite-dimensional object, the computation of such a model with aggregate risks is difficult. My model provides a tractable tool to analyze the fluctuations of aggregate economy and income distribution and assess policies over the business cycle under the setting of heterogeneous agents because it allows for exact aggregation, or equivalent to say, I can find a representative agent to study the behaviors of all agents.

This paper is closer to the recent studies embedding aggregate shocks into the framework with idiosyncratic investment risks. Goldberg (2014) sets up a framework with idiosyncratic investment risks in the business cycle. My paper differs from his in a few ways. First, I model incomplete markets with a unique non-state-contingent bond being the only security in the market; Goldberg provides state-contingent promises but with moral hazard. Second, he aims to build on a theoretical framework which incorporates uninsured idiosyncratic investment risks, aggregate shocks and borrowing constraint while I build up this model to help answer questions about policy analysis. Another example is the work of Nezafat and Slavik (2015), who construct a production-based asset pricing model with aggregate risks, idiosyncratic investment risks and financial frictions to explain the high volatility of asset prices. I, instead, talk about welfare and the assessment of fiscal policies.

The paper also belongs to the literature on fiscal policy. These studies usually either assume a representative agent and explore the effect of policy on the macroeconomy, neglecting the distributive issues; or they do consider heterogeneity but exclude aggregate risks, which makes them unable to discuss how effectively a government modifies fiscal policy to improve welfare when witnessing aggregate economic fluctuations. The exceptions are Werning (2007) and Bassetto (2014). Both employ complete markets to simplify the solution to the optimal policy problem. In contrast, I build up the problem in incomplete markets, in line with empirical evidence. In addition, Werning (2007) focuses on whether to smooth the tax rate while Bassetto (2014) builds up a model without capital so that the model is unable to assess the capital income tax. I focus on when to tax capital and issue debt based on the cyclicality of fiscal policy.

The rest of the paper is organized as follows. I extend Angeletos (2007)'s framework to model business cycles in Section 2 and characterize equilibrium in Section 3. In Section 4, I quantitatively study the income shares of different income groups and their cyclical properties generated by my model. I also make a comparison to the data. Section 5 carries out experiments on fiscal policy. Section 6 examines the robustness of policy evaluation with a constant labor tax and a consumption tax, constant idiosyncratic investment risks and without idiosyncratic investment risks. Section 7 concludes.

2.2 The Model

2.2.1 Economy

The economy is populated with two types of agents: entrepreneurs and workers. I normalize the measure of entrepreneurs to 1 and the measure of workers by λ. I interpret λ as the worker-entrepreneur ratio. There is a continuum of individuals for each type, indexed by i and j, respectively. Each worker is endowed with $\dfrac{1}{\lambda}$ unit of time so that the aggregate labor endowment is 1. I denote u_t as instantaneous utility; each agent maximizes her expected lifetime utility subject to her own budget constraint and borrowing constraint. The aggregate productivity z_t affects the production of each firm in the economy and follows an AR(1) process

$$\log z_{t+1} = \rho_z \log z_t + \epsilon^z_{t+1}, \tag{2.1}$$

where ϵ^z_{t+1} is normally distributed, $\epsilon^z_{t+1} \sim \mathcal{N}(0, \sigma_z^2)$, and the autoregressive parameter $\rho_z \in [0, 1)$.

2.2.2 Entrepreneurs

I assume that entrepreneurs only care about consumption. I specify a CRRA utility function $u(c^i_t) = \frac{(c^i_t)^{1-\gamma}}{1-\gamma}$, where γ denotes the degree of risk aversion. Each entrepreneur owns a private firm. At t, an entrepreneur can invest capital k^i_{t+1} in the firm owned by herself, but not in other private firms; she can buy or sell riskfree government bonds b^i_{t+1} as an alternative financial asset. Entrepreneurs do not work and receive asset gains as the unique source of income. r^i_t and R_t denote the return to her firm, and the interest rate of bonds, respectively, while τ^a_t, the capital tax rate. The effective wealth consists of gross income from capital and bond holdings net of taxation. The budget constraint and nonnegativity constraints are

$$c_t^i + k_{t+1}^i + b_{t+1}^i = [(1-\tau_t^a)(r_t^i - \delta) + 1]k_t^i + [(1-\tau_t^a)(R_t - 1) + 1]b_t^i, \tag{2.2}$$

$$c_t^i \geq 0 \text{ and } k_{t+1}^i \geq 0.$$

Entrepreneurs are allowed to borrow in the bond market but the borrowing amount has to fulfill the No-Ponzi condition

$$\lim_{T\to\infty} \mathbb{E}_t \frac{b_{t+T}^i}{\prod_{s=0}^{T-1}[(1-\tau_{t+s}^a)(R_{t+s} - 1) + 1]} = 0. \tag{2.3}$$

2.2.3 Firms

Each firm hires labor in a competitive labor market, employs its owner's capital and produces consumption goods. I name the firm run by the entrepreneur i also with i. I assume the neoclassical production technology

$$y_t^i = F(k_t^i, n_t^i, A_t^i) = (A_t^i)^\alpha (k_t^i)^\alpha (n_t^i)^{1-\alpha},$$

which exhibits constant returns to scale with respect to k and n. The firm-level productivity specific to firm i, A_t^i, affects the final output y_t^i. Particularly, A_t^i consists of two components, an idiosyncratic production risk e_t^i and aggregate productivity z_t: $\log A_t^i = \log z_t - \frac{\sigma_{e,t}^2}{2} + e_t^i$, where e_t^i is independently and identically distributed among firms and across time while z_t captures the business cycle. The idiosyncratic risk, e_t^i, is modelled as a normal, $e_t^i \sim \mathcal{N}(0, \sigma_{e,t}^2)$, where $\sigma_{e,t}^2$ represents the variance of idiosyncratic investment risks at t, which may vary across periods following $\sigma_{e,t}^2 = \sigma_e^2 \exp(-\eta \log z_t)$. $\eta > 0$ denotes the response of volatility to the cycle. Bloom et al (2014) measure the dispersion of TFP shocks for a panel of plants and find that the volatility of firm-specific productivity rises during the recession. The idiosyncratic risk captures the position of a specific firm ranked by the firm-level productivity. The correction term, $-\frac{\sigma_{e,t}^2}{2}$, renders the average productivity of private firms equivalent to the aggregate productivity. I define the profit of firm i as the firm revenue net of labor costs

$$\pi_t^i(k_t^i, n_t^i, A_t^i) = y_t^i - w_t n_t^i,$$

where w_t represents the wage rate at t. The competitive labor market ensures a universal wage rate in each period and the wage depends on the aggregate productivity and aggregate allocations.

The assumption of i.i.d idiosyncratic investment risks may seem restrictive. Nevertheless, it is necessary for tractability. Also, it may not be that unrealistic given that DeBacker et al (2013) find that business income is much less persistent than labor income and it is characterized by higher probabilities of extreme upward or downward mobility. For instance, conditional on leaving the starting business income decile, a household faces a 52% probability of moving to either of the two immediately adjacent deciles over a year. More strikingly, households starting at the lowest decile of the business income distribution face a 12% probability of transitioning to decile 8 or higher, whereas the corresponding probability is essentially zero for labor income. Hence, business income may transition from low to high amounts within a relatively short time and i.i.d shock is a reasonable abstraction from the fact.

2.2.4 Workers

Worker j has preferences on consumption, which is specified as $u(c_t^j) = \frac{(c_t^j)^{1-\gamma}}{1-\gamma}$. I assume that a worker shares the same curvature of consumption as an entrepreneur. Workers supply their labor in the competitive labor market, work in firms owned by entrepreneurs and consume all of their earnings in the manner of hand-to-mouth agents. This is not unrealistic given that quite a number of households hardly own any wealth other than a house. Workers provide identical working hours but differ in their labor efficiency e_t^j, which is independently and identically distributed across workers. A worker's personal labor income depends on idiosyncratic labor efficiency and the wage rate. A worker is taxed by a proportional labor tax rate, τ_t^n. The assumption of hand-to-mouth implies that the aggregate consumption and income for workers do not depend on the distribution of workers. I assume that idiosyncratic labor efficiency follows a persistent stochastic process,

$$\log e_{t+1}^j = \rho_w \log e_t^j + \epsilon_{t+1}^j, \tag{2.4}$$

where $\rho_w \in [0, 1)$ denotes the autocorrelation of labor efficiency and ϵ_{t+1}^j is normally distributed, $\epsilon_{t+1}^j \sim \mathcal{N}(0, \sigma_w^2)$.

The budget constraint for worker j is

$$c_t^j = (1 - \tau_t^n) w_t \frac{1}{\lambda} e_t^j. \tag{2.5}$$

2.2.5 Government

Government spending, g_t, is assumed to be exogenous, following an AR(1) process.

$$\log g_{t+1} = (1 - \rho_g) \log \bar{g} + \rho_g \log g_t + \epsilon_{t+1}^g, \tag{2.6}$$

where the steady state level of government consumption is $\bar{g}$, the autoregressive parameter $\rho_g \in [0, 1)$ and ϵ_{t+1}^g is normally distributed, $\epsilon_{t+1}^g \sim \mathcal{N}(0, \sigma_g^2)$.

Each period the government finances government spending by levying the proportional taxes $\{\tau_t^n, \tau_t^a\}$ and issuing bonds $\{B_{t+1}\}$.

$$\int_i \tau_t^a [(r_t^i - \delta) k_t^i + (R_t - 1) b_t^i] + \int_j \tau_t^n w_t \frac{1}{\lambda} e_t^j + B_{t+1} = g_t + R_t B_t, \tag{2.7}$$

where B_{t+1} denotes the total amount of bonds issued at t and paid off at $t+1$. I assume that the government does not distinguish the gains from risky assets and from riskfree assets so that it taxes the asset income at the same rate. In this paper, I intend to analyze the optimal policy of capital taxation and debt conditional on the long-run levels, or the effect on the social welfare of cyclical properties of taxation and debt. Thus I assume that fiscal policy τ_t^a and B_{t+1} are functions of log-deviation of output, indicating that the government adjusts its policy over the cycle,

$$\log\left(\frac{\tau_t^a}{\bar{\tau}^a}\right) = m_{Y\tau} \log\left(\frac{Y_t}{\bar{Y}}\right); \tag{2.8}$$

$$\log\left(\frac{B_{t+1}}{\bar{B}}\right) = \rho_B \log\left(\frac{B_t}{\bar{B}}\right) + m_{YB} \log\left(\frac{Y_t}{\bar{Y}}\right). \tag{2.9}$$

$\bar{\tau}^a$ represents the steady state level of tax rate, $\bar{Y}$ denotes the steady state value of output and $\bar{B}$ shows the steady state level of debt. $m_{Y\tau}$ and m_{YB} are two coefficients governing the responses of current tax rates and debt to output fluctuations, respectively. ρ_B denotes the autoregressive property of government debt[1]. For instance, the government

[1] I set the autoregressive specification of debt because the government has to stabilize the debt holding. As for tax policy, I choose a simple version of Leeper (1991)'s specification to facilitate my study.

taxes more capital income and issues more government bonds during recessions if $m_{Y\tau} < 0$ and $m_{YB} < 0$. To sum up, the government sets the rules of debt and capital income tax, and applies labor tax to balance the budget.

2.2.6 Timing

Every period aggregate shocks and idiosyncratic risks first hit the economy. The government then announces fiscal policy based on the current state. After observing the shocks and policy, workers supply labor while the firm optimally chooses the demand of labor. The firm takes labor and predetermined capital as inputs to produce. The entrepreneur consumes, accumulates capital and purchases bonds after she receives income from interest and bond payment, and pays taxes. The worker consumes all the after-tax labor income.

2.2.7 Stationarity

The current model would imply a nonstationary distribution of wealth and income. I add an exogenous death probability, Pr_d, to all entrepreneurs to guarantee limited income by entrepreneurs. Specifically the death shock is assumed to happen after every entrepreneur makes her own decision each time. Some entrepreneurs die while the same number of newborns enter into the economy. Every newborn inherits the average amount of risky and riskfree assets previously owned by the dead. I will show in the later section that entrepreneurs make saving and portfolio choices independent of the income distribution, only as functions of the aggregate state. Hence this assumption does not change the original choices before death and facilitates the computation.

2.3 Equilibrium

2.3.1 Equilibrium Definition

Denote K_t, B_t, z_t, g_t as the aggregate state S_t, and k_t^i, b_t^i, e_t^i as the individual state for entrepreneurs s_t^i.

Definition. *An equilibrium is a stochastic sequence of prices* $\{w_t(S_t), R_{t+1}(S_t)\}_{t=0}^{\infty}$*, a stochastic sequence of individual allocations* $\{c_t^i(s_t^i; S_t), k_{t+1}^i(s_t^i; S_t), b_{t+1}^i(s_t^i; S_t)\}_{t=0}^{\infty}$*,* $i \in$

$[0, 1]$, *for entrepreneurs,* $\{c_t^j(e_t^j; S_t), n_t^j(e_t^j; S_t)\}_{t=0}^{\infty}$, $j \in (1, \lambda+1]$, *for workers,* $\{y_t^i(s_t^i; S_t), n_t^i(s_t^i; S_t)\}_{t=0}^{\infty}$ *for firms, and aggregate allocations* $\{C_t^E(S_t), C_t^W(S_t), K_{t+1}(S_t), Y_t(S_t)\}_{t=0}^{\infty}$, *such that*

(1) Given prices $\{w_t(S_t), R_{t+1}(S_t)\}_{t=0}^{\infty}$, *fiscal policy* $\{\tau_t^n(S_t), \tau_t^a(S_t), B_{t+1}(S_t)\}$ *and the distribution of initial assets* k_0^i *and* b_0^i, *every entrepreneur* i *and every worker* j *maximize their respective lifetime utility by choosing* $\{c_t^i(s_t^i; S_t), k_{t+1}^i(s_t^i; S_t), b_{t+1}^i(s_t^i; S_t)\}_{t=0}^{\infty}$ *and* $\{c_t^j(e_t^j; S_t), n_t^j(e_t^j; S_t)\}_{t=0}^{\infty}$, *and every firm* i *maximizes its profit by choosing* $\{n_t^i(s_t^i; S_t), y_t^i(s_t^i; S_t)\}_{t=0}^{\infty}$.

(2) Aggregation: $C_t^E(S_t) = \int_i c_t^i(s_t^i; S_t)$, $Y_t(S_t) = \int_i y_t^i(s_t^i; S_t)$, $K_{t+1}(S_t) = \int_i k_{t+1}^i(s_t^i; S_t)$ *and* $C_t^W(S_t) = \int_j c_t^j(e_t^j; S_t)$ *for all* t.

(3) Labor market clearing: $\int_i n_t^i(s_t^i; S_t) = \int_j n_t^j(e_t^j; S_t)e_t^j$ *for all* t.

(4) Bond market clearing: $\int_i b_{t+1}^i(s_t^i; S_t) = B_{t+1}(S_t)$ *for all* t.

(5) Goods market clearing: $C_t^E(S_t) + C_t^W(S_t) + K_{t+1}(S_t) + g_t = Y_t(S_t) + (1-\delta)K_t(S_{t-1})$.

(6) The government budget constraint holds given capital tax and bond specifications for all t, *i.e.* (2.7), (2.8) *and* (2.9) *hold.*

For the expository purpose, I will mostly drop the state in each variable for the rest of my paper and only use the expression with the state when it is necessary.

2.3.2 Individual Behavior

Because the firm chooses employment, n_t^i, after observing the shock and after determining the capital stock, n_t^i is the only control variable to maximize the profit. By constant returns to scale, optimal firm employment and profit are linear in capital, as in Angeletos (2007):

$$n_t^i = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} A_t^i k_t^i = n(A_t^i, w_t)k_t^i, \tag{2.10}$$

$$\pi_t^i = \left[\alpha\left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}-1} A_t^i\right] k_t^i = r(A_t^i, w_t)k_t^i. \tag{2.11}$$

It indicates that the firm experiences linear returns to investment by adjusting its employment linearly to the capital stock. Notice that the return to a private firm i, $r_t^i = r(A_t^i, w_t)$, is expressed by the firm specific productivity, A_t^i, and the economy-wide wage which can be further expressed by the aggregate productivity and total capital stock.

Denote the effective wealth of entrepreneur i in period t by

$$x_t^i \equiv \left[(1-\tau_t^a)(r(A_t^i, w_t) - \delta) + 1\right] k_t^i + [(1-\tau_t^a)(R_t - 1) + 1] b_t^i.$$

I rewrite the budget constraint as $c_t^i + k_{t+1}^i + b_{t+1}^i = x_t^i$.

I characterize the entrepreneur's behavior following Angeletos (2007) in Lemma, whose proof is provided in Appendix A.

Lemma. *Given prices and fiscal policy, optimal consumption, capital stock in the private firm, and bond holdings are linear in effective wealth as shown in* (2.12), (2.13) *and* (2.14)*;*

$$c_t^i = \nu_t x_t^i, \tag{2.12}$$

$$k_{t+1}^i = (1-\nu_t)\phi_t x_t^i, \tag{2.13}$$

$$b_{t+1}^i = (1-\nu_t)(1-\phi_t) x_t^i, \tag{2.14}$$

where the marginal propensity to consume out of effective wealth, ν_t, and the share of private equity in the portfolio, ϕ_t, are two stochastic coefficients, depending solely on the current aggregate states, S_t, and satisfying

$$\phi_t = \underset{\phi \in [0,1]}{\arg\max}\, \mathbb{CE}_t \left\{ \phi_t[(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] + (1-\phi_t)[(1-\tau_{t+1}^a)(R_{t+1} - 1) + 1] \right\}, \tag{2.15}$$

$$\begin{aligned} \nu_t^{-\gamma} = \beta_s (1-\nu_t)^{-\gamma} \mathbb{E}_t \Big\{ \nu_{t+1}^{-\gamma} \{ \phi_t [(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] + \\ + (1-\phi_t)[(1-\tau_{t+1}^a)(R_{t+1} - 1) + 1] \}^{1-\gamma} \Big\}, \end{aligned} \tag{2.16}$$

where $\mathbb{CE}$ represents the certainty equivalent of an entrepreneur.[2]

Define the value function for entrepreneurs as $V(x_t^i)$ which is given by

$$V(x_t^i) = \frac{\nu_t^{-\gamma}(x_t^i)^{1-\gamma}}{1-\gamma}. \tag{2.17}$$

The entrepreneur makes the optimal portfolio choice, ϕ_t, by maximizing risk-adjusted portfolio returns, expressed by the certainty equivalent, $\mathbb{CE}$, of the portfolio return given the saving choice, $(1-\nu_t)$. If the return to capital is surely greater than the return to bonds, she will invest all her savings on capital. The uncertainty of capital return, though,

[2] I denote $\beta_s = \beta(1 - Pr_d)$

induces her to divide her investment on both assets, which ensures an interior point of portfolio decisions. The current specification of tax policy implies that a change in the capital tax rate fails to directly influence the portfolio choice since both types of assets are taxed at the same rate. Nonetheless, the capital tax rate distorts the saving choice, indirectly affecting asset returns and the composition of portfolio. Given the portfolio choice, the entrepreneur chooses the marginal propensity to consume according to the intertemporal condition to maximize the lifetime utility. Because of i.i.d idiosyncratic investment risks and homogeneity of the production and utility functions, entrepreneurs make the saving and portfolio decisions independent of income distribution. Hence the marginal propensity to consume and the risky-asset-over-wealth ratio only depend on the aggregate state.

The hand-to-mouth worker consumes all her after-tax labor income every period. So the consumption path is completely characterized by the budget constraint of workers.

2.3.3 General Equilibrium

Recall that aggregate productivity z_t and idiosyncratic risks e_t^i are orthogonal. Aggregate labor demand and firm profit are given by $N_t^D = \tilde{n}(w_t, z_t)K_t$ and $\Pi_t = \tilde{r}(w_t, z_t)K_t$, where

$$\tilde{n}(w_t, z_t) \equiv \int_{-\infty}^{\infty} n(A_t^i, w_t)\,\mathrm{d}F(e_t^i) = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} z_t,$$

and

$$\tilde{r}(w_t, z_t) \equiv \int_{-\infty}^{\infty} r(A_t^i, w_t)\,\mathrm{d}F(e_t^i) = \alpha\left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}-1} z_t.$$

$F(\cdot)$ represents the distribution function of i.i.d idiosyncratic investment risks. Thus $\tilde{n}(w_t, z_t)$ denotes the average labor employed by a firm and $\tilde{r}(w_t, z_t)$, the average capital return to a private firm. Let $N_t^S = \int_j \frac{1}{\lambda} e_t^j = 1$ represent aggregate labor supply; labor-market clearing requires that $N_t^D = \tilde{n}(w_t, z_t)K_t = 1$. The wage is expressed as $w_t = w(K_t, z_t) = (1-\alpha)\left(z_t K_t\right)^{\alpha}$. Further algebra shows that $r_t = \tilde{r}(w_t, z_t) = \alpha z_t^{\alpha} K_t^{\alpha-1}$. I write aggregate output as $Y_t = \Pi_t + w_t N_t^S = r_t K_t + w_t = z_t^{\alpha} K_t^{\alpha}$. Aggregate allocations are also independent of the wealth distribution because consumption, bond holdings, and private investment are linear in individual wealth and idiosyncratic risks in every period are i.i.d across firms and periods. The general equilibrium is determined by the following equations, where I drop the determination of z_t and g_t as they are assumed to be exogenous. I denote C_t^E as the aggregate consumption assigned to the group of entrepreneurs and C_t^W, to the group of workers.

$$C_t^E + C_t^W + K_{t+1} + g_t = z_t^\alpha K_t^\alpha + (1-\delta)K_t, \tag{2.18}$$

$$C_t^W = (1-\tau_t^n)w_t, \tag{2.19}$$

$$C_t^E = \nu_t\{[(1-\tau_t^a)(r_t-\delta)+1]K_t + [(1-\tau_t^a)(R_t-1)+1]B_t\}, \tag{2.20}$$

$$K_{t+1} = (1-\nu_t)\phi_t\{[(1-\tau_t^a)(r_t-\delta)+1]K_t + [(1-\tau_t^a)(R_t-1)+1]B_t\}, \tag{2.21}$$

$$B_{t+1} = (1-\nu_t)(1-\phi_t)\{[(1-\tau_t^a)(r_t-\delta)+1]K_t + [(1-\tau_t^a)(R_t-1)+1]B_t\}, \tag{2.22}$$

These five equations, together with (2.15) and (2.16), solve for seven variables C_t^E, C_t^W, K_{t+1}, B_{t+1}, R_{t+1}, ν_t and ϕ_t, given the current aggregate state and fiscal policy.

2.3.4 Computation Process

The framework allows me to separate the computation of aggregate variables and income distribution. First, I apply the second-order approximation to simulate a time series of equilibrium at the aggregate level over the business cycle starting from the steady state. I am able to accomplish it because, economically speaking, every entrepreneur makes exactly identical saving and portfolio decisions regardless of their individual wealth. In another word, I can always find a representative entrepreneur to study the behavior of all entrepreneurs. Then I divide aggregate allocations into the individual level. The next period's capital and bonds of entrepreneur i can be calculated as a product of saving choice, portfolio choice and effective wealth as (2.13) and (2.14). The first two components are obtained from the time series of general equilibrium. I separate the last part into idiosyncratic capital returns, interest rate of bonds and capital and bonds stock held from last period. The time series of equilibrium contains the values of interest rate and the average capital returns, then from (35) in Appendix B we get idiosyncratic capital returns by inputting a stochastic process of idiosyncratic risks. Thus the amount of assets forms a recursion given asset prices in the aggregate level and idiosyncratic risks. Workers share the total labor income determined by general equilibrium with different labor efficiency.

I simulate 2000 entrepreneurs and, as a consequence, 10000 is the number of workers. I allocate equally the steady state level of capital and bonds to 2000 entrepreneurs as the initial point. Since individual capital returns are idiosyncratic to entrepreneurs, lucky entrepreneurs accumulate more and more assets so that the gap of capital income rises over time. Yet the assumption of random death prevents infinite inequality. I record the income of every agent, including entrepreneurs and workers, order all values and then compute quantile statistics and the like. I run a simulation with 2500 periods and drop the first 1500 periods to guarantee stability.

2.4 Business Cycle Statistics of the Income Distribution

This section starts from calibration. Then the section investigates the behavior of aggregate variables over the business cycle. It further shows that the model, with aggregate shocks, can mimic the income distribution and its cyclicality at a small computational cost. The model matches the dynamics of the income distribution shown in the data qualitatively for most the quantiles.

2.4.1 Calibration

Numerical analysis is needed to evaluate the effect of idiosyncratic investment risks on the steady state. This subsection first describes the details of calibrating the model.

I assume that a period in my model corresponds to a year. I use some parameter values which are common in the literature of business cycle: The capital share of output, α, is assumed to be 0.36; the depreciation rate, δ, 0.08.

The probability of death, Pr_d, for entrepreneurs is interpreted to match the working lifetime of an entrepreneur or a private firm. However, the statistic is unclear and hard to measure. I assume the probability of death to be 0.025 for an expected working lifetime of 40 years. I then calibrate the discount rate so that the subjective discount rate of entrepreneurs after considering death produces a realistic capital-output ratio, 2.7, as in the US. I assume the worker-entrepreneur ratio as 5 to match the estimation of business owners and self-employed in the US reported by Cagetti and De Nardi (2006).[3]

[3]The value seems much higher than the estimated proportion of hand-to-mouth workers. For instance,

The choice of risk aversion degree matters for my study, especially when I evaluate fiscal policy. The baseline calibration sets the risk aversion degree equal to 2. The reason is that to assess fiscal policy requires the welfare comparison and the value is commonly applied in the literature on welfare.

I apply the effective marginal tax rate of the US computed by Devereux et al. (2008) as the capital tax rate in my model. I take the mean of these rates from 1979 to 2005 and the value is 22.19%, which is close to the capital tax rate used by Chari et al. (1994). I pick the labor income tax rate to balance the government budget. In the benchmark calibration, I set the steady state levels of debt as 60 percent of output and government consumption as 18 percent of output, both of which depict the U.S. situation before the recent crisis.

It is hard to find an exact measurement of idiosyncratic investment risks in the empirical studies. DeBacker et al. (2012) find that the standard deviation of uninsurable idiosyncratic income risks from privately held businesses accounts for 45 percent of the average business income.[4] Therefore I calibrate my model to match their finding; specifically the cross sectional standard deviation of individual firm's return is 45% of the average return. Notice that a number of macroeconomic studies, such as Sandri (2014), apply the volatility of firms' returns as 50% of the mean return, thus I assure my calibration close to their choice and my results comparable.

I set the autocorrelation of idiosyncratic labor income risks equal to 0.9989 and the standard deviation, 0.0166 using the estimation of Storesletten et al. (2004).

Kaplan et al. (2014) define a household to be hand-to-mouth in a period if it consumes all its cash-on-hand for the period and carries zero liquid wealth between the current and next period. They conclude that on average, 31 percent of US households are hand-to-mouth from 1989 to 2010. The assumption of hand-to-mouth workers in this paper aims to shut down saving from workers who face labor income risks to simplify the analysis and the computation. Since the model does not involve housing, I focus on the individuals who have no non-housing wealth defined by net worth. About 60% of households in the US own no wealth other than their home according to the 2007 Survey of Consumer Finances. Although my choice of the worker/entrepreneur ratio still overstates the composition of hand-to-mouth workers in this sense, I have a reason as follows. The 2016 poll by The Associated Press-NORC Center for Public Affairs Research uncovers that two thirds of Americans would have difficulty coming up with the money to cover a 1000-dollar emergency. In a 2015 study by the Federal Reserve, 47 percent of respondents said they either could not cover even a 400-dollar emergency expense or would have to sell something or borrow money. This paper mainly investigates the fluctuations of income distribution in the business cycle. When a sufficiently negative shock happens, individuals probably face a much larger loss than 1000 dollars so that the assumption of more than 80 percent of hand-to-mouth workers in my model does not exaggerate much the situation of US citizens over the business cycle.

[4] They employ individual income tax returns data from the Internal Revenue Service over 23 years, compute for each household the time series standard deviation of its business income normalized by the household's average total income over time, and then combine those business income "coefficients of variation" into one cross sectional average.

The calibration for the exogenous technology process and government spending follows Chari et al. (1994). I use the FRED dataset and the OECD tax dataset to pin down the responses of capital tax and debt to output in the above specified fiscal policy. I regress debt on the HP-filtered log of output and lagged debt following the fiscal policy specification. I estimate the response of capital tax rate to output deviation with a similar method except that the regressor only contains HP-filtered log of output. Worth to mention, the estimated tax policy has little correlation with output. Meanwhile, the government reduces debt in the expansion and vice versa.

Recall that I correlate the volatility of idiosyncratic investment risks with aggregate productivity, $e_t^i \sim \mathcal{N}(0, \sigma_e^2 \exp(-\eta \log z_t)$. I calibrate the sensitivity of the variance of idiosyncratic investment risks to aggregate productivity shocks, η, to match the empirical finding of Bloom et al. (2016) that plant-level TFP shocks increased in variance by 76% during the recent recession (2008 to 2009) compared to the years before the recession (2005 to 2006). I plug the deviation of the logarithm of TFP from the trend in 2006 and in 2009 into the expression of the variance of idiosyncratic investment risks and back out the value of η. All above parameter values for the benchmark calibration are included in Table 2.1.

2.4.2 Cyclical Behavior of Aggregate Variables in General Equilibrium

Before presenting the results on the income distribution, I first show the cyclical behavior of aggregate variables in general equilibrium to better understand the mechanism of countercyclical idiosyncratic investment risks. Figure 2.1 plots impulse response functions of selected variables after one-standard-deviation negative productivity shock hits the economy for the baseline model, for the model with constant idiosyncratic investment risks ($\eta = 0$), and for the model without idiosyncratic risks. Particularly, I emphasize the marginal propensity to save, $(1-\nu)$, and the portfolio choice, ϕ, since these two variables summarize the key property of the entire equilibrium allocations and prices.

The marginal propensity to save, $(1-\nu)$, rises after the adverse shock only in the baseline model. Investors tend to save more assets in the recession because highly countercyclical idiosyncratic investment risks amplify the precautionary saving motives. In addition, they invest more in safe assets so that the capital share in the portfolio, ϕ,

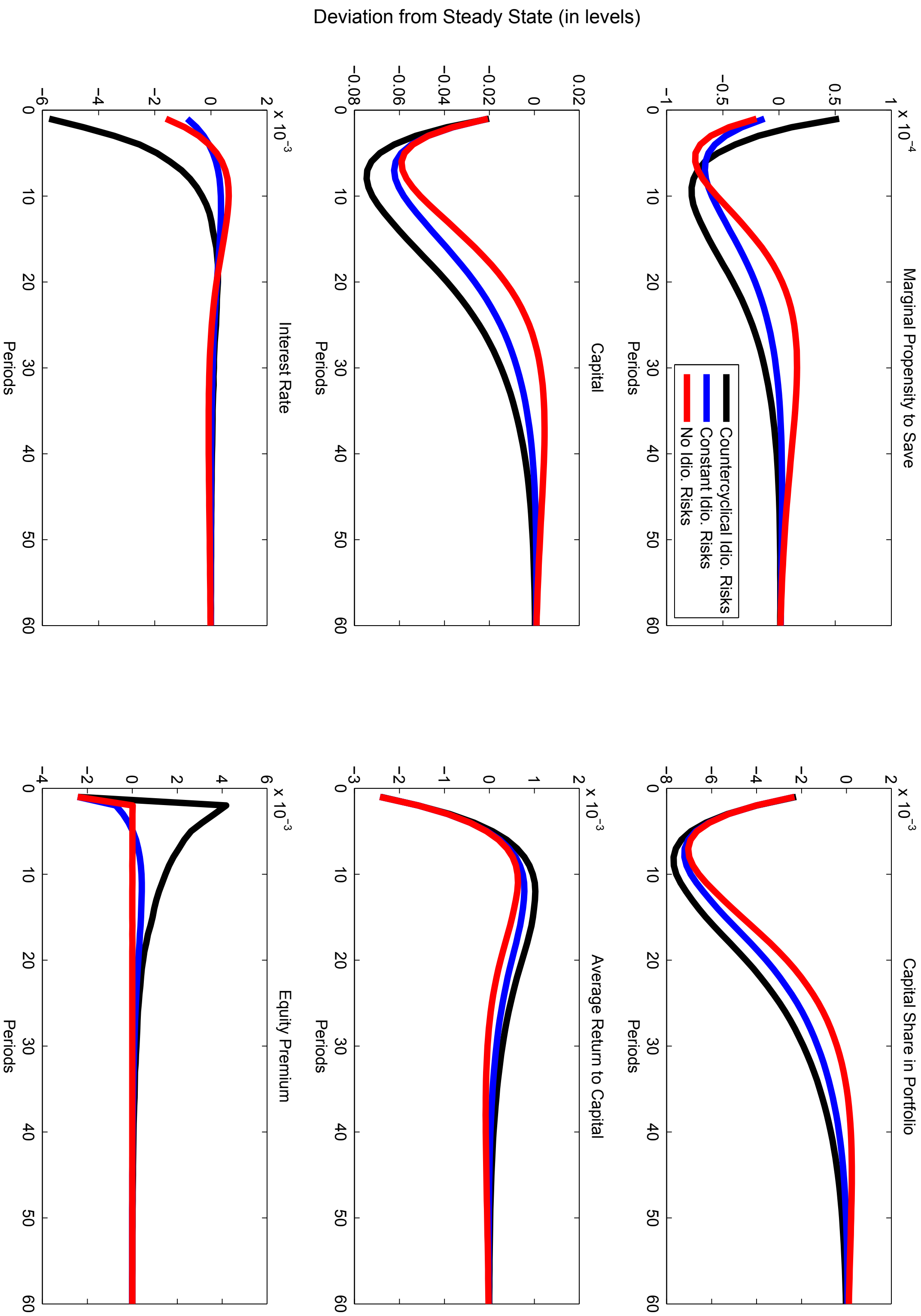


Figure 2.1: Impulse Responses

Table 2.1: Calibration for the benchmark case

Parameters	**Values**	**Descriptions**
α	0.36	capital share of output
β	0.98	subjective discount rate
Pr_d	0.025	probability of death for entrepreneurs
δ	0.08	depreciation rate
γ	2	risk aversion degree
$\bar{b}$	60% output	steady state level of bonds
$\bar{g}$	18% output	steady state level of government consumption
$\bar{\tau}^a$	22.19%	steady state asset income tax rate
σ_e	45% average return	standard deviation of idiosyncratic risks
λ	5	worker/entrepreneur ratio
ρ_w	0.9989	autocorrelation of labor efficiency
σ_w	0.0166	cross-sectional standard deviation of labor income
η	11.10	sensitivity of the variance of idiosyncratic production risks to productivity shocks
ρ_z	0.81	autocorrelation of technology process
σ_z	0.05	standard deviation of technology shock
ρ_g	0.89	autocorrelation of government spending
σ_g	0.07	standard deviation of government spending shock
ρ_B	0.84	autocorrelation of government debt
m_{YB}	-0.62	response of debt to output gap
$m_{Y\tau}$	-0.077	response of tax to output gap

shows procyclicality, which causes procyclical capital stock.

The cyclical behavior of interest rate is the consequence of the movement of government bonds' supply and demand over the cycle. Agents demand more riskfree assets in the bust; meanwhile, the government supplies more bonds according to the policy specification. A smaller increase in supply relative to the change in demand results in a higher equilibrium price of government bonds, or equivalently, a lower interest rate, especially with countercyclical idiosyncratic investment risks. The average return to capital shows an identical pattern in all three cases, following the change in productivity. With countercyclical idiosyncratic risks, equity premium increases in the second period and remains positive for around 30 periods in the recession, although the average return to capital and interest rate both decline.

Entrepreneurs expect a higher return to capital compared to bonds so that in the baseline model, the capital share in the portfolio declines less than the other two models at the early stage after shock. Yet it recovers to the steady state more slowly because high variance of idiosyncratic investment risks in the recession is persistent.

2.4.3 Income Inequality: Level

The income of an entrepreneur i is defined as the asset gains, $I_t^i = (r_t^i - \delta)k_t^i + (R_t - 1)b_t^i$, as Castañeda et al (1998) do, while the income of a worker j is the labor income, $\frac{1}{\lambda} w_t e_t^j$. My model does a decent job in generating income equality as the data show. For comparison, I take the statistics of the US income distribution from Current Population Survey and compute the means of income shares over time. Next column reports the means of income shares of different income groups obtained from the simulation. My model fits the data in the sense that it generates considerable income inequality although the rich obtain less income and the poor share more compared with the data.

Table 2.2: Income Inequality: Level

Quantiles	**Share of income (Data)**	**Share of income (Model)**
1st (bottom 20%)	4.1%	6.96%
2nd (20-40%)	10.3%	14.72%
3rd (40-60%)	16.4%	18.23%
4th (60-80%)	23.9%	22.51%
5th (80-95%)	27.0%	22.40%
Top 5%	18.4%	15.18%
Ratios	**Value (Data)**	**Value (Model)**
95/50 ratio	3.29	2.08
50/20 ratio	2.40	1.42

All the values of income shares are means computed using corresponding data from CPS from 1947 to 2013. CPS provides the statistics of households only since 1967 so I pick up the income share of families from 1947 to 1966.

Entrepreneurs account for the majority in the top income group and for over one third of agents in the bottom as shown in Table 2.3. It follows the dispersed distribution of idiosyncratic investment risks. Most entrepreneurs are rich in the model, yet some unlucky ones find themselves in the low income groups. Since entrepreneurs in the benchmark calibration are in line with the definition of business owners and self-employed, the simulation generates poor entrepreneurs as in reality since part of self-employed have low income. It indicates that income inequality in the baseline model is obtained by modelling entrepreneurs with relatively high idiosyncratic investment risks, which is confirmed by other research, such as Benhabib et al. (2011) and Nirei and Aoki (2016). In addition, I assume hand-to-mouth workers, which creates heterogeneity in the behaviors of agents and leads to inequality.

Table 2.3: Proportion of Entrepreneurs

Quantiles	Model
1st (bottom 20%)	34.04%
2nd (20-40%)	6.06%
3rd (40-60%)	5.99%
4th (60-80%)	7.91%
5th (80-95%)	16.67%
Top 5%	67.34%

2.4.4 Income Inequality: Cyclical Behavior

Table 2.4 reports the correlations with output of the income shares owned by income groups in the data and in my model. I choose the same data from CPS as for the level of income inequity and choose seasonally adjusted real GDP measured by 2009 billion dollars during 1948-2013. I detrend the shares, ratios and the log of real GDP by HP-filter and calculate the correlations. For the model, I pick the time series of income shares of the corresponding income groups and obtain the correlations with the output over the business cycle.

Table 2.4: Correlation of Income Shares with Output

Income groups	Data	Model
bottom 20%	0.29	0.80
20-40%	0.30	0.96
40-60%	0.15	0.87
60-80%	-0.19	0.66
80-95%	-0.44	-0.36
Top 5%	0.07	-0.86
Ratios	**Data**	**Model**
95/50 ratio	-0.24	-0.86
50/20 ratio	-0.06	-0.82

All the values of income shares use corresponding data from CPS from 1947 to 2013. CPS provides the statistics of households only since 1967 so I pick up the income share of families from 1947 to 1966.

My model with the baseline calibration qualitatively matches correlation with output of income shares except the top 5% group and the 60-80% group although the model still differs from the data quantitatively. My model overstates the procyclicality for groups from the bottom to 60% and the countercyclicality for 80-95%.

I wonder why my model presents such quantitative behaviors of the income distribution over the cycle. To uncover the mechanism, I build up two more statistics to measure the impact of business cycle on individual income: coefficients of variation of $t+1$'s income for entrepreneurs and workers given t's information, $\text{Coef.Var.}_t(I^i_{t+1})$ and $\text{Coef.Var.}_t(I^j_{t+1})$. These two statistics aim to show how an aggregate shock at $t+1$ affects $t+1$'s income for agents who expect it at t. Notice that for simplicity I do not consider the death risk in the subsequent discussion of mechanism since to add death does not affect any qualitative result. I express the two statistics as follows, whose derivation lies in Appendix C.

$$\text{Coef.Var.}_t(I^i_{t+1}) = \frac{\phi_t \alpha z_t^{\alpha\rho_z} K_{t+1}^{\alpha-1}\sqrt{\exp\left[\sigma_e^2 \exp(-\eta\rho_z \log z_t) + \frac{1}{2}(2\alpha - \eta\sigma_e^2)^2\sigma_z^2\right] - \exp\left(\alpha^2\sigma_z^2\right)}}{\phi_t \alpha z_t^{\alpha\rho_z} \exp\left(\frac{1}{2}\alpha^2\sigma_z^2\right) K_{t+1}^{\alpha-1} + (1-\phi_t)(R_{t+1}-1)}. \tag{2.23}$$

$$\text{Coef.Var.}_t(I^j_{t+1}) = \sqrt{\exp\left(\alpha^2\sigma_z^2 + \sigma_w^2\right) - 1}. \tag{2.24}$$

The conditional coefficient of variation of workers' next period income is constant. It results from the assumption of hand-to-mouth workers. The wage inherits the cyclical behavior from the output; in particular, its coefficient of variation equates the one of output. Idiosyncratic labor income risks with constant variance enlarge the total variance of workers' income so that eventually the summed volatility appears in the coefficient of variation, indicating the function of both aggregate shocks and idiosyncratic labor income risks on the cross-sectional disparity of workers' income.

When the high productivity is realized, the wage rate rises. The mean labor income become higher. The lucky workers in the high state experience a larger increase in their income while the unlucky workers see a smaller increase or even a decrease in the income, depending on the size of idiosyncratic risks. Indeed, almost all workers are highly likely to obtain higher income, though the extent varies, in the high state because idiosyncratic labor income risks have a much smaller dispersion than aggregate shocks in the benchmark simulation. The low productivity produces opposite results compared to the preceding ones. Since the coefficient of variation of workers' income remains constant over the cycle, the difference of income in the low state for rich and poor workers remains the same as in the high state. Summarizing both states I conclude that income of all workers shows procyclical behavior.

The conditional coefficient of variation of entrepreneurs' income over the cycle looks more complicated. I start from the discussion on the denominator or, roughly speaking, the conditional mean of income. Idiosyncratic investment risks, as they are assumed to be i.i.d, exert no effect on next period's expected income. Only aggregate shocks influence the conditional expectation through the channel of returns to assets. When a negative shock happens at t, persistence implies probably low productivity next period, resulting in smaller share of capital in the portfolio, lower capital stock and lower expected returns to capital. As a consequence, the conditional expected income diminishes during the recession.

When investigating the cyclical property of conditional standard deviation, recall that I assign a positive value to the response of volatility to aggregate productivity, η. An adverse aggregate shock causes an increase in the value inside the root operation while the product of other terms outside the root operation goes down. Therefore, the expression cannot easily judge the moving direction of the whole nominator and of the fraction facing an aggregate shock.

Instead, I analyze the variation of entrepreneurs' income over the cycle numerically by considering three cases: when the economy is in the steady state and when the economy suffers a negative and a positive shock. Figure 2.2 plots the cumulative distribution function for entrepreneurs when one standard deviation of negative or positive aggregate shock hits the economy and when the economy stays in steady state. The low state enlarges the income gap between rich and poor entrepreneurs since idiosyncratic investment risks rise in the recession. The increased individual difference magnifies the damage of low state to the poor, and reduces it to the rich. The expansion witnesses a similar change but with an opposite direction and a smaller scale. Thus the possibility of obtaining lower income in the expansion is much less than that of higher income in the recession. To sum up, the income share of rich entrepreneurs exhibits countercyclicality while that of poor entrepreneurs shows procyclicality.

When I add up the income distribution of workers and entrepreneurs, the bottom income group consists of poor workers and poor entrepreneurs. Thus, income share of poor

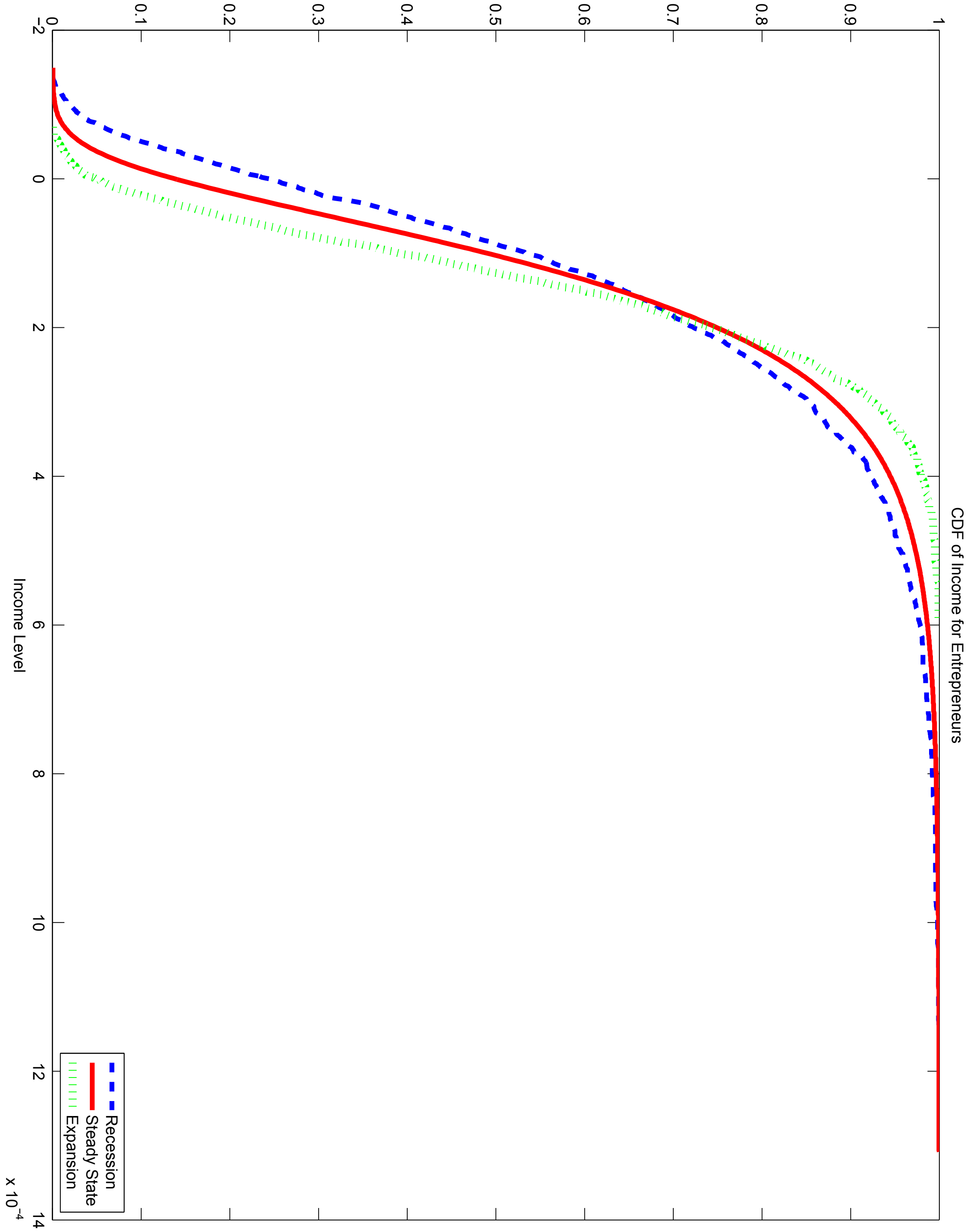


Figure 2.2: CDF of Entrepreneurs in Recession, Normal Time and Expansion

agents must show procyclicality. Entrepreneurs account for a large number of top earners, so rich agents in my model see countercyclical income share. The sign of correlations of income shares in the middle groups is indeterminate, but less countercyclical than the top end and less procyclical than the bottom end. It also explains why cyclicality of income shares generally decreases with the income.

2.5 Fiscal Policy Experiments

This section explores if heterogeneity impacts the choice of fiscal policy from a normative perspective. The government, fixing the steady state level of fiscal policy, adjusts the cyclicality of policy instruments to maximize the welfare of different agents. I put emphasis on the time to tax capital more and the time to issue more debt. I find a welfare conflict between entrepreneurs and workers. Under the baseline calibration, the government should tax capital more and issue more debt in the recession. However, the welfare conflict implies that the government may cut the capital income tax rate in the recession if it gives more welfare weight to workers, which overturns the finding of Chari et al. (1994).

2.5.1 Objective and Method of Policy Experiments

As the welfare criterion, I adopt the ex-ante utilitarian social utility in which the social planner sums up the life-time individual utility across agents,

$$U_0^{SP} = \int_i \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \int_j \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \frac{(c_t^j)^{1-\gamma}}{1-\gamma} = \int_i V(x_0^i) + \int_j V(x_0^j).$$

I denote the value function of entrepreneurs and workers as $\int_i V(x_0^i)$ and $\int_j V(x_0^j)$, respectively. Recall that the value function of entrepreneurs is showed as (2.17),

$$V(x_0^i) = \frac{\nu_0^{-\gamma} x_{i,0}^{1-\gamma}}{1-\gamma}.$$

Given that the initial conditions of capital and bond holdings are identical under different policy specifications, the value function of entrepreneurs maps one-to-one to the marginal propensity to consume. Moreover, since the risk aversion degree, γ, is larger than one in my welfare analysis, the life-time utility for entrepreneurs increases with the marginal propensity to consume. In another word, more consumption conditional on a certain available wealth heightens the welfare of entrepreneurs.

Besides, Appendix D derives the value function of workers and I replicate the result:

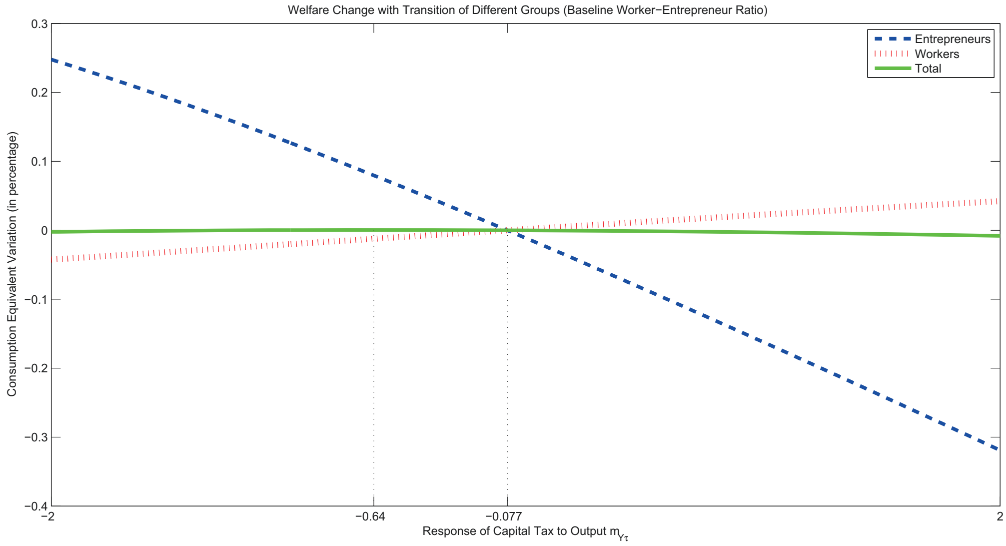
$$V(x_0^j) = \lambda^\gamma \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left\{ \frac{[(1-\tau_t^n)w_t]^{1-\gamma}}{1-\gamma} \right\} \exp\left(\frac{\sigma_w^2 (1-\gamma)^2}{2(1-\rho_w^2)} \right).$$

The government searches for the time to tax capital more and to issue more debt by changing the responses of tax rate and of debt to output, and comparing the social welfare. The economy starts from a fixed state at which state variables are evaluated at their long-run means with baseline calibration. Thus the government does consider the effect of a transition from the baseline policy to a new one on the welfare in this exercise. I compute the consumption equivalent variation in percentage of the aggregate life-time utility across entrepreneurs, across workers and across agents, as the response of tax rate to output and the response of debt to output range from -2 to 2 and from -1 to 1, respectively, under the baseline calibration of other parameters previously used to produce the dynamics of income distribution. The base to compute the consumption equivalent variation is the life-time utility under the baseline policy to match the policy data, for each type of agents and the counterpart of social welfare.

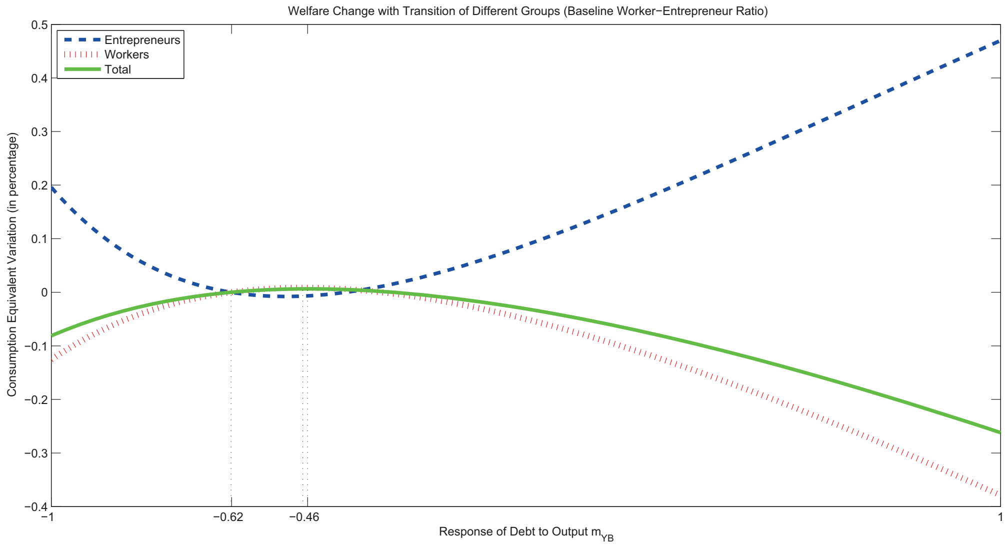
2.5.2 Welfare Conflict between Entrepreneurs and Workers

Figure 2.3a shows the welfare change in terms of the consumption equivalent variation of entrepreneurs, workers and all agents under different capital tax policy when the debt policy is fixed. Figure 2.3b, instead, depicts the welfare change of different groups under various debt policy if the government fixes the capital tax policy. They both manifest that entrepreneurs and workers favor different policy. In the recession, entrepreneurs prefer a low debt level and a high capital tax rate while workers, a high debt level and a low capital tax rate. Figure 2.8 in Appendix E plots the welfare change if the government simultaneously chooses two fiscal policy instruments. It also indicates a welfare conflict between entrepreneurs and workers under various combinations of fiscal policy instruments.

To better compare the effect of the cyclicality of fiscal policy, I pick the combinations of the responses of capital tax rate and debt to output which maximize the welfare of



(a) Welfare Change as $m_{Y\tau}$ Changes (Baseline Worker-Entrepreneur Ratio)



(b) Welfare Change as m_{YB} Changes (Baseline Worker-Entrepreneur Ratio)

Figure 2.3: Welfare Change of Different Groups

entrepreneurs, workers and all agents, respectively. Table 2.5 reports the welfare change of different agents and the change in aggregate variables under three policy combinations. All the values are percentage changes compared to the counterparts under the benchmark calibration.

If the government aims to maximize the welfare of entrepreneurs, a high capital tax rate and a low debt level should be chosen in the recession. The mean level of bonds rises while the average capital tax rate declines compared to the benchmark calibration. More government bonds give entrepreneurs insurance and crowd out capital, yet less tax burden encourages entrepreneurs to invest more in capital. The combination of policy turns out to slightly raise capital stock, which furthermore pushes up the wage. The mean consumption of entrepreneurs increases due to less tax burden and higher asset income. The government budget balance requires a higher average labor tax rate, which causes a decrease in consumption of workers.

The volatility of consumption of entrepreneurs increases by more than 160% compared to the baseline policy. Yet the standard deviation of consumption of entrepreneurs still accounts for less than 5 % of the mean. For entrepreneurs, the increase in the mean consumption outweighs the welfare loss from a higher volatility. Workers see less mean consumption and a higher volatility. Thus, entrepreneurs are better off and workers are worse off.

The policy combination that maximizes the welfare of workers features the debt negatively correlated with output and the capital tax rate positively correlated with output. Under this policy, the mean level of bonds diminishes while the average capital tax rate increases. The distortionary capital income tax disincentivizes entrepreneurs to invest, leading to lower average capital stock and a lower mean wage. The mean consumption of entrepreneurs decreases after entrepreneurs are taxed more heavily and deprived of considerable amount of riskfree assets. The current policy, however, results in higher mean consumption of workers because they suffer less from labor taxation. Although the consumption of workers varies over the cycle more than under the baseline policy, the mean effect again dominates the fluctuation effect. Therefore, only workers are better off.

2.5.3 Time to Tax Capital and Increase Debt

Chari et al. (1994) investigate optimal fiscal policy in the business cycle. They find that the correlation of an optimal capital income tax rate with technology shock is negative with uncontingent debt in their baseline model. In addition, when there is a negative innovation to the technology shock or a positive innovation to government consumption, there is a positive innovation in the tax on private assets. They argue that it is efficient for these shocks which affect the government budget constraint to be absorbed mainly by the tax on private assets.

The policy that maximizes the social welfare in my model features a 0.45 percentage-point increase in the capital tax rate and a 0.37 percent increase in the debt level when the output drops by one percent. The policy combination results in a low average level of debt and a low average capital tax rate. Capital stock and wage increase by 0.44% and 0.14%, respectively, compared to the baseline policy. The policy also leads to a higher labor tax rate. The entrepreneurs have more average consumption while workers have slightly less consumption. The volatility of the consumption of entrepreneurs and workers see opposite changes; in particular, the consumption of entrepreneurs fluctuates by a much larger margin. The mean effect dominates the fluctuation effect for entrepreneurs. The large decrease in the volatility of consumption of workers compensates partially the negative effect of the reduced mean consumption. Therefore, entrepreneurs are better off while workers are worse off; but compared to the policy preferred by entrepreneurs, the welfare loss of workers is much smaller.

The result seems to confirm the finding of Chari et al. (1994). Yet the capital rate under the policy combination maximazing social welfare increases in the recession less than under the one favoring only entrepreneurs. It implies that considering workers' welfare lowers the capital tax that should be levied in the bust. The utilitarian social utility weighs less the equality compared to many other welfare criteria. The government would specify a capital tax rule in which the rate increases little or even decreases in the recession if the government puts sufficient emphasis on workers. Therefore, it is possible that the government should cut the capital tax rate in the recession.

2.5.4 Impulse Responses of Aggregate Variables under Different Policy

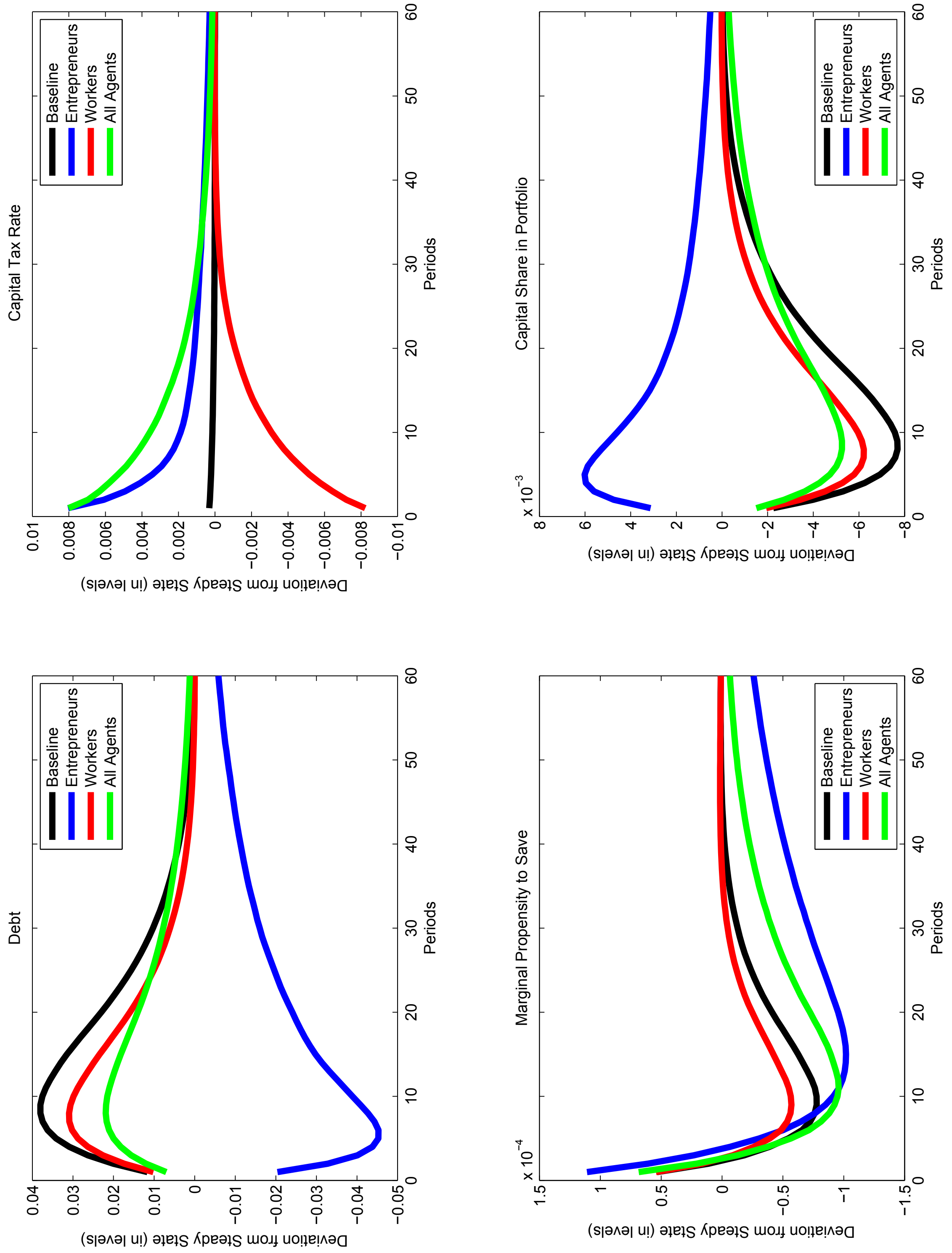
Figure 2.4 plots the impulse responses of aggregate variables to one-standard-deviation negative aggregate productivity shock under the baseline calibration and the other three policy combinations optimizing entrepreneurs, workers and all agents, respectively.

The dynamics of fiscal policy instruments after a negative productivity shock differ with different policy targets. The policy which maximizes the welfare of entrepreneurs requires a low debt level while other policy specifications indicate higher debt than in the normal time. The difference in the change of debt causes the qualitative disparity in the change of portfolio choice. All four policy combinations discussed lead to higher marginal propensity to save in the recession as a result of countercyclical investment risks. Only under the policy maximizing entrepreneurs' welfare do entrepreneurs hold more capital. The equity premium increases, although the average return to capital and interest rate both decline, because entrepreneurs tend to hold safe assets after confronting much higher idiosyncratic investment risks in the recession. Generally speaking, the less debt when the adverse shock hits, the more capital and output in the early stage after the shock. However, the recovery of capital slows down and falls behind the counterparts under other policy combinations because the average return to capital is lower. Entrepreneurs see a lower level of consumption under each policy combination; among all them, the entrepreneur-preferred policy pulls down consumption most and causes the slowest recovery.

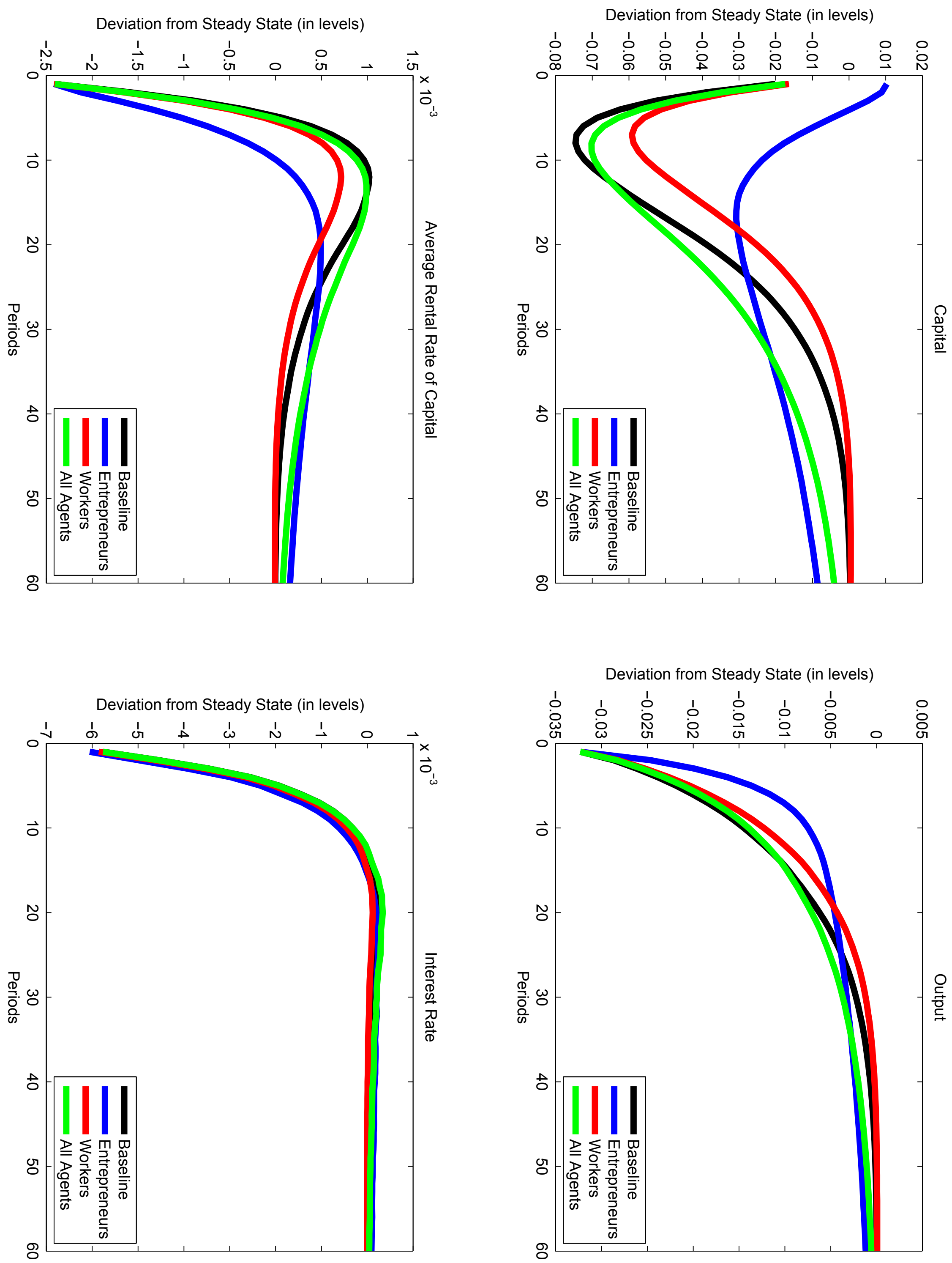
Low output and low wage result from low productivity. The labor tax rate rises merely under the entrepreneur-preferred policy because under this policy the government has to reduce the debt holding and the budget balance constraint requires higher tax revenues. It lowers the consumption of workers more than other policy. Yet since the policy maximizing entrepreneurs' welfare also raises the capital stock in the first several periods after the shock, output and wage recover fast. In addition, the labor tax rate drops rapidly to even a lower level than the steady state. Thus, the consumption of workers returns to the steady state within short periods.

2.6 Robustness Check

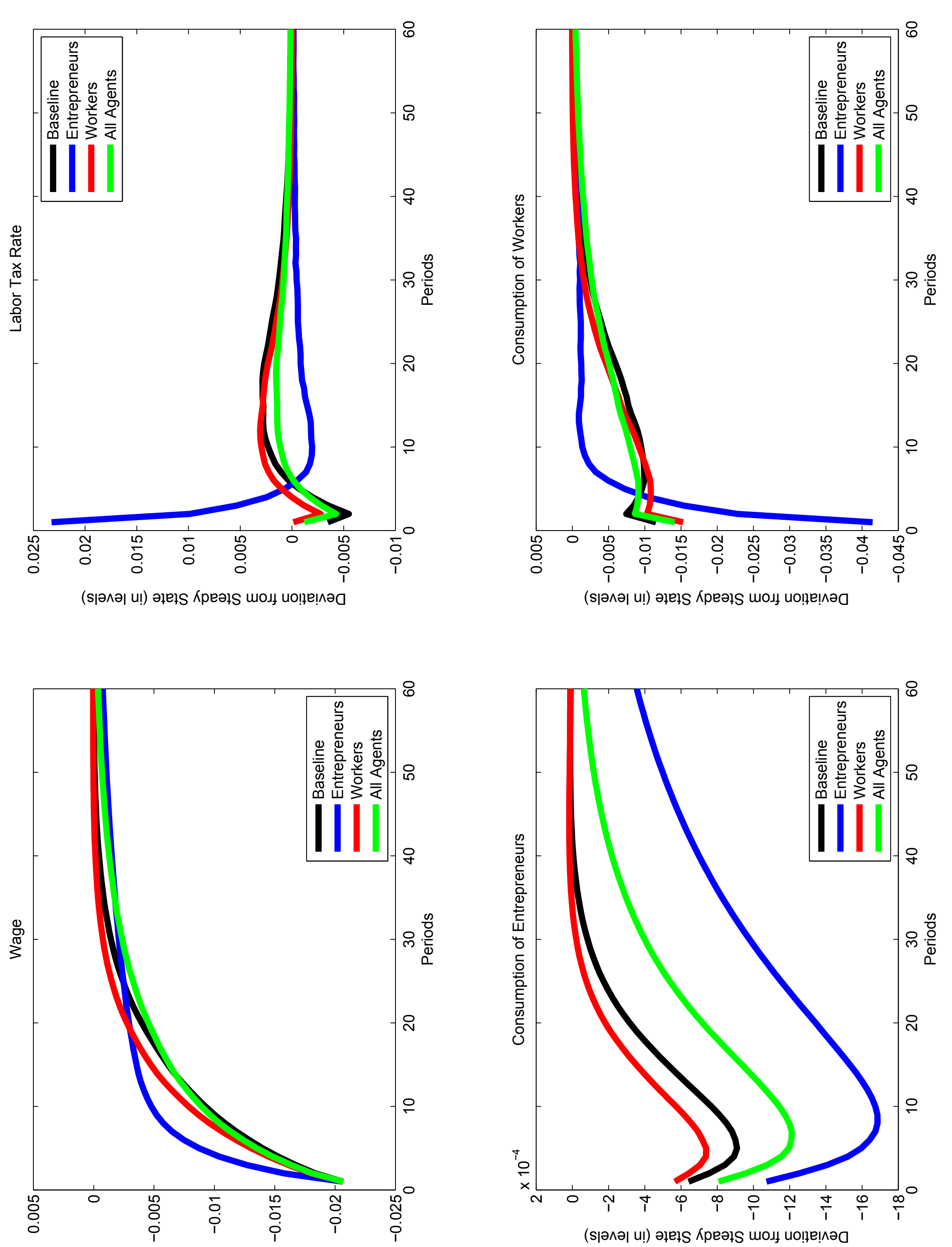
This section examines the robustness of the result of welfare conflict. First, the government turns to only change the fiscal policy instruments that directly affect entrepreneurs. Second, I let the volatility of idiosyncratic investment risks uncorrelated to the aggregate productivity, $\sigma_{e,t}^2 = \sigma_e^2$, and simulate the welfare of heterogeneous agents under different



(a) Impulse Responses of Selected Variables



(b) Impulse Responses of Selected Variables (Continued)



(c) Impulse Responses of Selected Variables (Continued)

Figure 2.4: Impulse Responses under Different Policy

policy. Lastly, I shut down the idiosyncratic investment risks, $\sigma_{e,t} = 0$, to see its effect on the welfare. The welfare conflict exists in the first exercise. Yet qualitatively the time to tax capital income no longer incurs welfare conflicts between groups. The welfare conflict disappears in the qualitative sense in the last two exercises.

2.6.1 Constant Labor Tax Rate and Consumption Tax on Entrepreneurs

The above section indicates that the change of cyclical behavior of fiscal policy redistributes resources between entrepreneurs and workers. Consequently, the welfare of entrepreneurs and workers witnesses different reactions. Since I use labor tax rate to balance the government budget, if the government raises debt or lowers capital tax rate, labor tax rate may have to rise, which directly lowers the consumption of workers. This direct effect may outweigh the indirect effect of wage on the welfare of workers. This subsection conducts an experiment in which the government levies a new consumption tax pointing only to entrepreneurs so that the adjustment of capital income tax rate only influences the welfare of workers through wage. The capital income tax and debt policy are regulated according to policy specifications, (2.8) and (2.9), while keeping constant level of labor tax rate but applying consumption tax rate to balance the government budget. The policy prescription of capital income tax, debt and consumption tax directly affects entrepreneurs' welfare through saving and portfolio choices. Besides, the government no longer taxes the income from riskfree assets, or government bonds, in this exercise. This modification aims to see if the choice of policy may depend on tax deductions on certain assets.

The government budget constraint now reads

$$\int_i \left[\tau_t^k \left(r_t^i - \delta\right) k_t^i + \tau_t^c c_t^i\right] + \int_j \tau^n w_t \frac{1}{\lambda} e_t^j + B_{t+1} = g_t + R_t B_t, \tag{2.25}$$

where τ_t^c represents consumption tax that varies over the cycle to balance the budget. Appendix F exhibits the solution to the modified entrepreneurs' problem. The fundamental result that saving and portfolio choices are independent of wealth distribution still holds. The capital income tax, however, directly influences the portfolio choice since the tax rate enters into the expression of certainty equivalent. I assume zero consumption tax in steady state so that the new model evaluated with the baseline parameter values matches the same calibration targets as the previous model.

I find that the welfare conflict still occurs as shown in Figure 2.5. The only qualitative difference in the preferences of fiscal policy lies in the choice on the cyclical property of debt policy. Entrepreneurs prefer a low debt level in the recession while workers, instead, prefer a slightly high debt policy. Workers, like entrepreneurs, favor a procyclical capital tax policy, although the capital tax rate favored by workers rises in the recession less than the one preferred by entrepreneurs. This is because a procyclical capital tax policy engenders higher capital stock and thus higher wage while workers no longer have to pay more taxes to fill the gap of government budget due to fewer tax revenues.

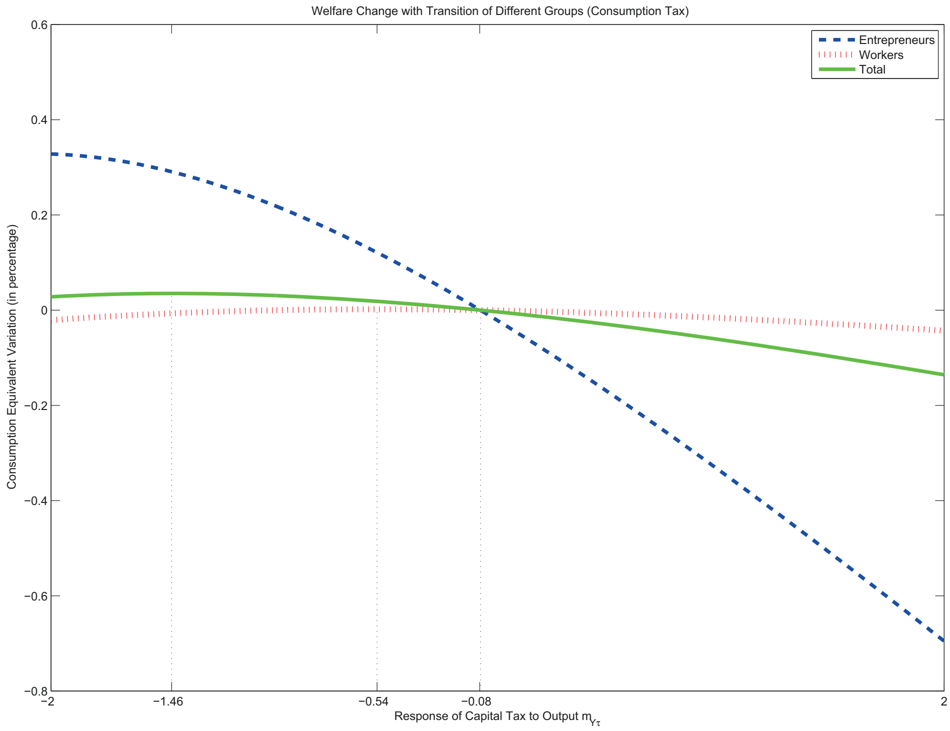
To summarize, the welfare conflict lies in the choice of both policy instruments. But entrepreneurs and workers only have different preferences in debt policy in the qualitative sense.

2.6.2 Constant Volatility of Idiosyncratic Investment Risks

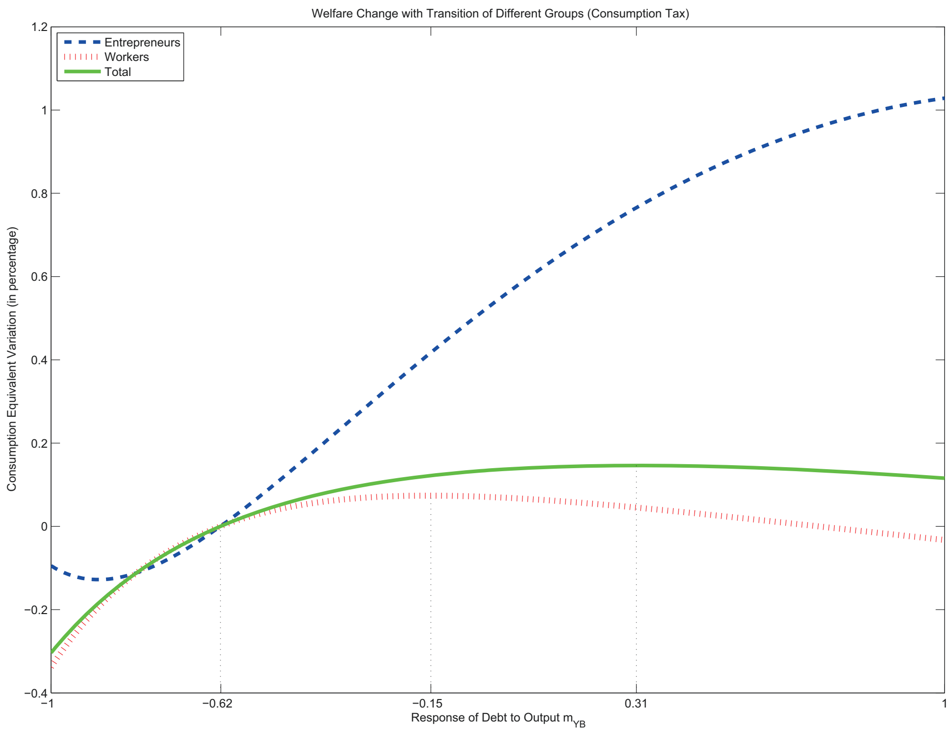
This subsection carries out the baseline policy experiment in which the government chooses both the responses of fiscal policy instruments to output except that the volatility of idiosyncratic investment risks is set to be constant, $\sigma_{e,t}^2 = \sigma_e^2$. Figure 2.6 plots the welfare change of entrepreneurs, workers and social utility, if the government only modifies the response of the capital tax rate to output, $m_{Y\tau}$, or if the government only changes the response of the debt to output, m_{YB}.

A comparison of Figure 2.3 and Figure 2.6 clearly shows a difference in the welfare change with countercyclical and constant idiosyncratic investment risks. First, countercyclical idiosyncratic investment risks enlarge the scale of welfare change. Second, countercyclical risks influence the cyclical choice of fiscal policy that aims to maximize the welfare of certain groups. With constant idiosyncratic investment risks, entrepreneurs prefer a higher tax rate in the recession, but much lower than the preferred policy with countercyclical idiosyncratic investment risks. Workers, unlike in the baseline case, choose an even higher tax rate than the one entrepreneurs favor in the recession. Meanwhile, entrepreneurs turn to a higher debt level in the recession, while workers prefer a relatively constant but still countercyclical debt policy.

In a nutshell, countercyclical idiosyncratic investment risks magnify the fluctuations of

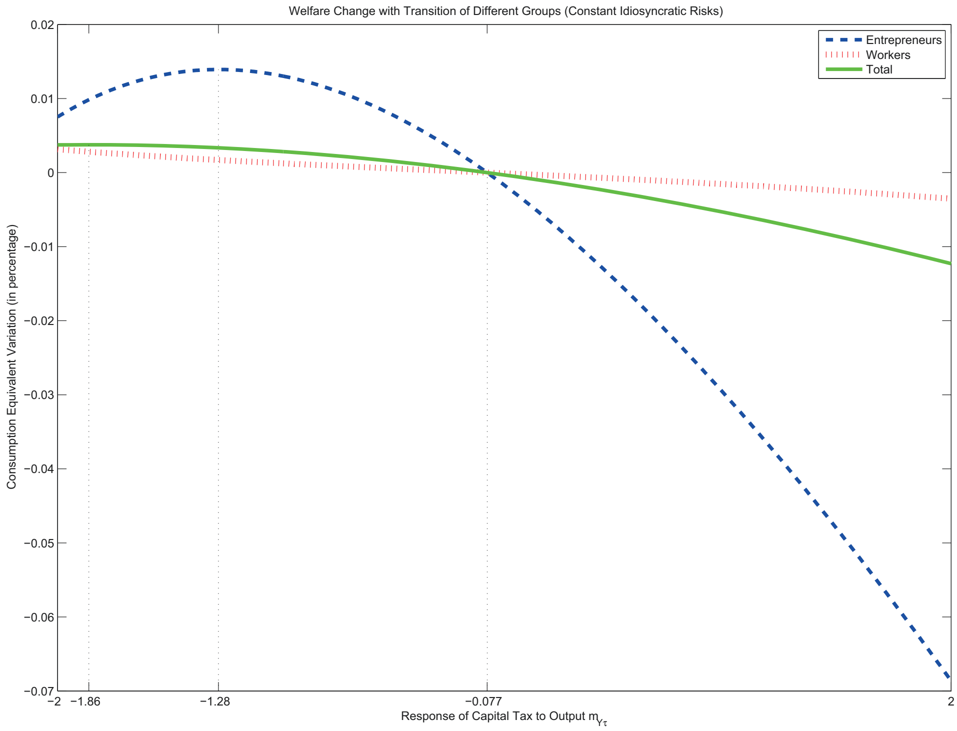


(a) Welfare Change as $m_{Y\tau}$ Changes (Consumption Tax)

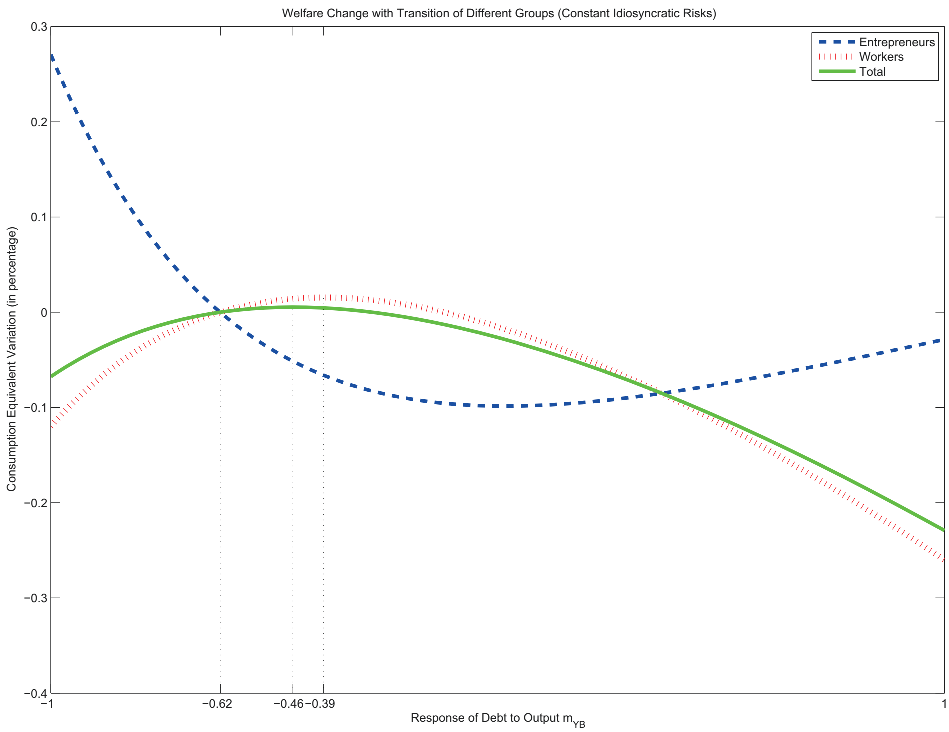


(b) Welfare Change as m_{YB} Changes (Consumption Tax)

Figure 2.5: Robustness Check: Consumption Tax



(a) Welfare Change as $m_{Y\tau}$ Changes (Constant Idiosyncratic investment risks)



(b) Welfare Change as m_{YB} Changes (Constant Idiosyncratic investment risks)

Figure 2.6: Robustness Check: Constant Idiosyncratic investment risks

capital and output; in particular, investors are far less likely to lose in the boom thanks to asymmetry of idiosyncratic investment risks in the boom and in the bust, thus investors are inclined to the tax policy that encourages more investment in the expansion. The asymmetry of risks disappears with constant idiosyncratic risks and the gap of expected return to capital in the expansion and in the recession shrinks. Therefore the fluctuation effect becomes relatively more important to investors, and as a result, they choose a less volatile tax policy and a countercyclical debt policy.

Workers prefer a procyclical tax policy with constant idiosyncratic investment risks because it raises the capital stock and wage. They have fewer concerns for higher mean of labor tax because of less volatility in the aggregate economy.

The result of the welfare conflict between entrepreneurs and workers no longer holds with constant idiosyncratic investment risks in the qualitative sense though different groups choose different quantitative responses of fiscal policy to output.

To sum up, countercyclicality of idiosyncratic investment risks matters for the choice of the cyclical property of fiscal policy.

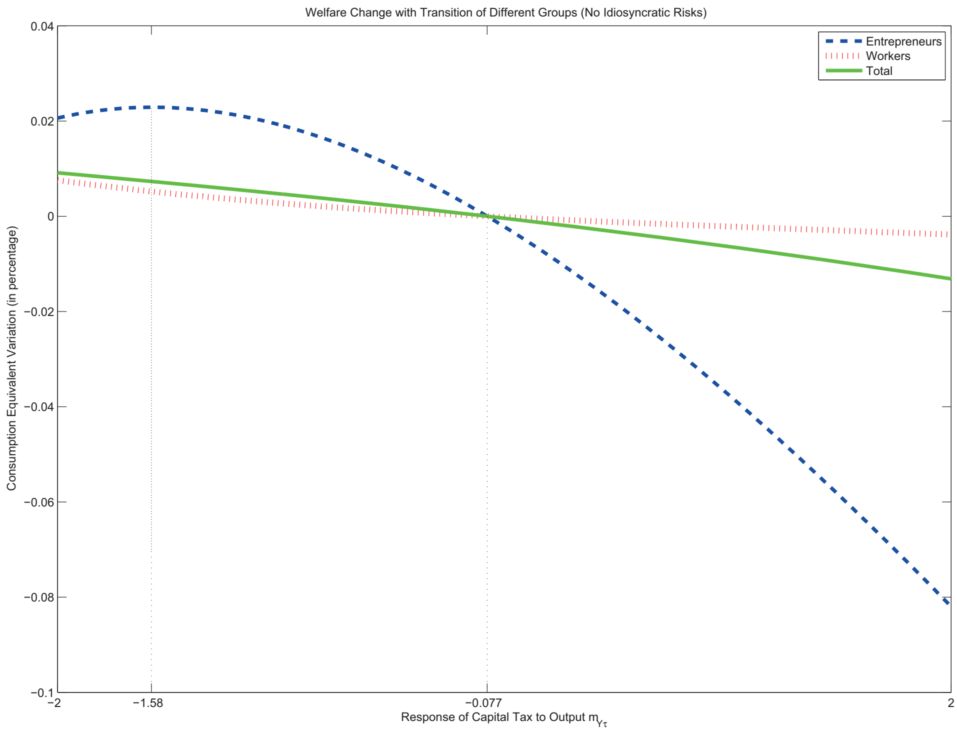
2.6.3 No Idiosyncratic Investment Risks

This subsection shuts down idiosyncratic investment risks and reassesses fiscal policies. Notice that the model features heterogeneity even without idiosyncratic risks because entrepreneurs and workers are assumed to receive income from different sources. Figure 2.7 plots the welfare change of entrepreneurs, workers and social utility under different capital tax policy if the debt policy is fixed, or under different debt policy if the capital tax policy is fixed.

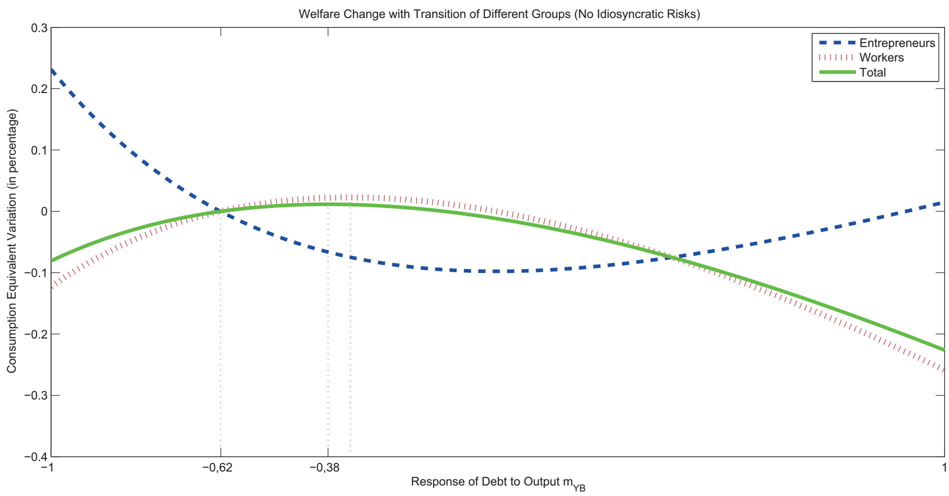
Figure 2.7 shows the same pattern in the welfare change of the three groups as with constant idiosyncratic investment risks. Qualitatively, all groups prefer a procyclical capital tax policy and a countercyclical debt policy.

2.7 Conclusion

This paper focuses on studying the effect of fiscal policies over the business cycle on investment and the welfare of heterogeneous agents under idiosyncratic investment risks, which is novel in macroeconomics. Idiosyncratic investment risks, although proven to



(a) Welfare Change as $m_{Y\tau}$ Changes (No Idiosyncratic investment risks)



(b) Welfare Change as m_{YB} Changes (No Idiosyncratic investment risks)

Figure 2.7: Welfare Change of Different Groups without Idiosyncratic investment risks

have a larger standard deviation than the one of idiosyncratic labor income risks, draw less attention and are underdeveloped, especially in the literature on business cycle. My study, together with a few others, pioneers merging idiosyncratic investment risks into the business cycle framework and applies the model to answer specific questions. I use the framework to first generate the level and dynamics of income distribution and then conduct several fiscal policy experiments.

My model takes into account three observations. First, wealth is highly concentrated among the rich. Second, wealth tends to be invested in an undiversified portfolio, even for the rich who in general have more investment options. Third, poor diversification in investment implies high idiosyncratic investment risks, which, furthermore, increase in the recession. The third observation is little considered in previous studies. I model it, associating idiosyncratic investment risks with aggregate productivity.

The model provides a tractable tool to analyze the dynamics of aggregate variables and income distribution, discuss the welfare change, and evaluate fiscal policy over the business cycle under the context of heterogeneous agents. The tractability results from the assumption of i.i.d idiosyncratic investment risks which render saving and portfolio independent of wealth distribution. All entrepreneurs behavior identically conditional on their wealth so that exact aggregation can be achieved.

The model matches the level and dynamics of income inequality as in the US data. The simulated income distribution features large income shares in top income groups and small income shares in bottom income groups as shown in the Current Population Survey data. As for the cyclical property of income distribution, the simulated result qualitatively matches most of the correlations of income shares with output, and 95/50 and 50/20 ratios.

I answer the question of when to tax capital income and when to issue more debt, both of which are rarely explored in the literature. The policy experiment emphasizes a welfare conflict between entrepreneurs and workers. The qualitative result on debt policy is robust to a modified model with a constant labor tax rate and a varying consumption tax levied only on entrepreneurs to balance the government budget. But the welfare conflict disappears after removing the countercyclicality of idiosyncratic investment risks or removing these risks at all.

Appendix

2.A Proof of Lemma 1

I start from showing the characterization of allocations and choices in the individual level.

The Euler equations derived from the entrepreneur optimization problem are

$$(c_t^i)^{-\gamma} = \beta_s \mathbb{E}_t \left\{ (c_{t+1}^i)^{-\gamma} [(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] \right\}, \tag{2.26}$$

$$(c_t^i)^{-\gamma} = \beta_s \mathbb{E}_t \left\{ (c_{t+1}^i)^{-\gamma} [(1-\tau_{t+1}^a)(R_{t+1} - 1) + 1] \right\}. \tag{2.27}$$

I guess that the solution to the entrepreneur optimization problem is as follows:

$c_t^i = \nu_t x_t^i$, and $k_{t+1}^i = (1-\nu_t)\phi_t x_t^i$, so $b_{t+1}^i = (1-\nu_t)(1-\phi_t)x_t^i$ from the budget constraint of the entrepreneur. I will determine later the coefficients ν_t and ϕ_t which only depend on the current aggregate state. To simplify the notation, I define the aggregate state at t, $S_t = (K_t, B_t, z_t, g_t)$. Then I write $\nu_t = \nu_t(S_t)$ and $\phi_t = \phi_t(S_t)$. With some algebra,

$$\begin{aligned} x_{t+1}^i &= \left[(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1\right] k_{t+1}^i + [(1-\tau_{t+1}^a)(R_{t+1} - 1) + 1] b_{t+1}^i \\ &= \left\{ \phi_t(S_t)[(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] + (1-\phi_t(S_t))[(1-\tau_{t+1}^a)(R_{t+1} - 1) + 1] \right\} \\ &\quad (1-\nu_t(S_t)) x_t^i. \end{aligned} \tag{2.28}$$

Then the first Euler equation becomes

$$\begin{aligned} \nu_t(S_t)^{-\gamma}(x_t^i)^{-\gamma} &= \beta_s \mathbb{E}_t \left\{ (\nu_{t+1}(S_{t+1}) x_{t+1}^i)^{-\gamma} [(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] \right\} \\ &= \beta_s (1-\nu_t(S_t))^{-\gamma} (x_t^i)^{-\gamma} \mathbb{E}_t \left\{ \nu_{t+1}(S_{t+1})^{-\gamma} [(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] \right. \\ &\quad \left\{ \phi_t(S_t)[(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] + (1-\phi_t(S_t))[(1-\tau_{t+1}^a)(R_{t+1} - 1) \right. \\ &\quad \left. \left. +1] \right\}^{-\gamma} \right\}. \end{aligned}$$

Then we cross out the same factors from both handsides,

$$\nu_t(S_t)^{-\gamma} = \beta_s(1-\nu_t(S_t))^{-\gamma}\mathbb{E}_t\left\{\nu_{t+1}(S_{t+1})^{-\gamma}[(1-\tau^a_{t+1})(r(A^i_{t+1},w_{t+1})-\delta)+1]\right.$$
$$\left.\{\phi_t(S_t)[(1-\tau^a_{t+1})(r(A^i_{t+1},w_{t+1})-\delta)+1]+(1-\phi_t(S_t))[(1-\tau^a_{t+1})(R_{t+1}-1)+1]\}\}^{-\gamma}\right\}. \tag{2.29}$$

Likewise, the second Euler equation can be transformed as

$$\nu_t(S_t)^{-\gamma} = \beta_s(1-\nu_t(S_t))^{-\gamma}\mathbb{E}_t\left\{\nu_{t+1}(S_{t+1})^{-\gamma}[(1-\tau^a_{t+1})(R_{t+1}-1)+1]\right.$$
$$\left.\{\phi_t(S_t)[(1-\tau^a_{t+1})(r(A^i_{t+1},w_{t+1})-\delta)+1]+(1-\phi_t(S_t))[(1-\tau^a_{t+1})(R_{t+1}-1)+1]\}^{-\gamma}\right\}. \tag{2.30}$$

Combining these two equations, we obtain the equality between the gross returns of risky and risk-free assets:

$$\begin{aligned}0 = {} & \mathbb{E}_t\left\{\nu_{t+1}(S_{t+1})^{-\gamma}(1-\tau^a_{t+1})(r(A^i_{t+1},w_{t+1})-\delta+1-R_{t+1})\right.\\
& \left.\{\phi_t(S_t)[(1-\tau^a_{t+1})(r(A^i_{t+1},w_{t+1})-\delta)+1]+(1-\phi_t(S_t))[(1-\tau^a_{t+1})(R_{t+1}-1)+1]\}^{-\gamma}\right\}.\\
= {} & \int_0^\infty\left(\int_0^\infty \nu_{t+1}(S_{t+1})^{-\gamma}(1-\tau^a_{t+1})(r(e^i_{t+1},\epsilon^z_{t+1},w(\epsilon^z_{t+1});z_t)-\delta+1-R_{t+1})\right.\\
& \{\phi_t(s_t)[(1-\tau^a_{t+1})(r(e^i_{t+1},\epsilon^z_{t+1},w(\epsilon^z_{t+1});z_t)-\delta)+1]+(1-\phi_t(S_t))[(1-\tau^a_{t+1})(R_{t+1}-1)+1]\}^{-\gamma}\\
& \left.\mathrm{d}F(e^i_{t+1})\right)\ \mathrm{d}F(\epsilon^z_{t+1}).\end{aligned} \tag{2.31}$$

Multiply the above Equations 26 and 27 with ϕ_t and $1-\phi_t$, respectively, and sum up to get:

$$\nu_t(S_t)^{-\gamma} = \beta_s(1-\nu_t(S_t))^{-\gamma}\mathbb{E}_t\left\{\nu_{t+1}(S_{t+1})^{-\gamma}\right.$$
$$\left.\{\phi_t(S_t)[(1-\tau^a_{t+1})(r(A^i_{t+1},w_{t+1})-\delta)+1]+(1-\phi_t(S_t))[(1-\tau^a_{t+1})(R_{t+1}-1)+1]\}^{1-\gamma}\right\}. \tag{2.32}$$

Define $\phi_t = \arg\max_{\phi\in[0,1]} \mathbb{CE}_t\left\{\phi_t[(1-\tau^a_{t+1})(r(A^i_{t+1},w_{t+1})-\delta)+1]+(1-\phi_t)[(1-\tau^a_{t+1})(R_{t+1}-1)\right.$ -

where

$$\mathbb{CE}_t\left\{\phi_t[(1-\tau^a_{t+1})(r(A^i_{t+1},w_{t+1})-\delta)+1]+(1-\phi_t)[(1-\tau^a_{t+1})(R_{t+1}-1)+1]\right\}$$
$$=\Bigg[\int_{-\infty}^{\infty}\Bigg(\int_{-\infty}^{\infty}\{\phi_t(S_t)[(1-\tau^a_{t+1})(r(e^i_{t+1},\epsilon^z_{t+1},w(\epsilon^z_{t+1});z_t)-\delta)+1]+$$
$$+\ (1-\phi_t(S_t))[(1-\tau^a_{t+1})(R_{t+1}-1)+1]\}^{1-\gamma}\,\mathrm{d}F(e^i_{t+1})\Bigg)\,\mathrm{d}F(\epsilon^z_{t+1})\Bigg]^{\frac{1}{1-\gamma}}. \tag{2.33}$$

The process requires numerical solutions. The preceding integral demonstrates that the optimal saving and portfolio choices depend only on the aggregate state. I drop the notation of aggregate state since now.

Given ϕ_t and τ^a_{t+1}, ν_t can be computed by (31).

Next I derive the functional form of the value function for entrepreneurs. I first guess that the value function $V(x^i_t)=\psi_t u(x^i_t)$. From the envelope theorem, $V'(x^i_t)=u'(c^i_t)$, which is

$$\psi_t(x^i_t)^{-\gamma}=(c^i_t)^{-\gamma}=[\nu_t(x^i_t)]^{-\gamma}$$

I simplify the above equation by crossing out the common factor so that

$$\psi_t=\nu_t^{-\gamma}. \tag{2.34}$$

Hence, $V(x^i_t)=\dfrac{\nu_t^{-\gamma}(x^i_t)^{1-\gamma}}{1-\gamma}$.

Now I verify whether the value function obtained fits the entrepreneur's optimization problem. Notice that $V(x^i_t)=\dfrac{(c^i_t)^{1-\gamma}}{1-\gamma}+\beta_s\mathbb{E}_tV(x^i_{t+1})$. I insert the expressions of the value function and consumption, Eq. (13), into the equation.

$$\frac{\nu_t^{-\gamma}(x^i_t)^{1-\gamma}}{1-\gamma}=\frac{[\nu_t(x^i_t)]^{1-\gamma}}{1-\gamma}+\beta_s\mathbb{E}_t\left[\frac{\nu_{t+1}^{-\gamma}(x^i_{t+1})^{1-\gamma}}{1-\gamma}\right]$$

I multiply both handsides by $(1-\gamma)$, remove the first term on the right handside to the left, and combine the two terms on the left handside,

$$
\begin{aligned}
&\nu_t(1-\nu_t)^{-\gamma}(x_t^i)^{1-\gamma} \\
&= \beta_s \mathbb{E}_t\left[(1-\nu_{t+1})^{-\gamma} x_{i,t+1}^{1-\gamma}\right] \\
&= \beta_s \mathbb{E}_t\Big[(1-\nu_{t+1})^{-\gamma}\big\{(1-\tau_{t+1}^a)[\phi_t(r(A_{t+1}^i, w_{t+1})+1-\delta)+ \\
&+\ (1-\phi_t)R_{t+1}]\nu_t(x_t^i)\big\}^{1-\gamma}\Big].
\end{aligned}
$$

Then I eliminate the common factors ν_t and $(x_t^i)^{1-\gamma}$ from both handsides,

$$
(1-\nu_t)^{-\gamma} = \beta_s \nu_t^{-\gamma} \mathbb{E}_t\left[(1-\nu_{t+1})^{-\gamma}\left\{(1-\tau_{t+1}^a)[\phi_t(r(A_{t+1}^i, w_{t+1})+1-\delta)+(1-\phi_t)R_{t+1}]\right\}^{1-\gamma}\right],
$$

which is validated by (31). $\square$

2.B Effect of idiosyncratic investment risks

This section shows how idiosyncratic investment risks affect the portfolio choice at the steady state. For the exposional convenience, I simply assume the risk aversion degree $\gamma = 1$, that is, the logarithm case.

From the wage determination $w_t = (1-\alpha)(z_t K_t)^{\alpha}$ and the specification of risks, I rewrite the expression of individual capital investment return

$$\begin{aligned} r^i_{t+1} &= \alpha \left(\frac{1-\alpha}{w_{t+1}}\right)^{\dfrac{1}{\alpha}-1} A^i_{t+1} \\ &= \alpha z^{\alpha-1}_{t+1} K^{\alpha-1}_{t+1} A^i_{t+1} \\ &= \alpha z^{\alpha}_{t+1} K^{\alpha-1}_{t+1} \exp\left[\frac{-\sigma^2_e}{2} + e^i_{t+1}\right] \\ &= \check{r}_{t+1} \exp\left(e^i_{t+1}\right), \end{aligned} \tag{2.35}$$

where $\check{r}_{t+1}$ denotes $\alpha z^{\alpha}_{t+1} K^{\alpha-1}_{t+1} \exp\left[\frac{-\sigma^2_{e,t+1}}{2}\right]$, showing risk-adjusted average return to capital in the economy.

$$\begin{aligned} &\mathbb{CE}_t \left\{\phi_t\left(r^i_{t+1} - \delta\right) + (1-\phi_t)(R_{t+1}-1)\right\} \\ \approx &\left(\int_{-\infty}^{\infty} \left[\phi_t\left(\check{r}_{t+1} - \delta\right) + (1-\phi_t)(R_{t+1}-1)\right]^{1-\gamma} + \frac{1-\gamma}{2}\sigma^2_{e,t+1}\right. \\ &\left\{-\gamma\left[\phi_t\left(\check{r}_{t+1} - \delta\right) + (1-\phi_t)(R_{t+1}-1)\right]^{-\gamma-1}\left(\phi_t \check{r}_{t+1}\right)^2 + \right. \\ &\left.\left. + \left[\phi_t\left(\check{r}_{t+1} - \delta\right) + (1-\phi_t)(R_{t+1}-1)\right]^{-\gamma} \phi_t \check{r}_{t+1}\right\} \mathrm{d}F\left(\epsilon^z_{t+1}\right)\right)^{\dfrac{1}{1-\gamma}} \end{aligned} \tag{2.36}$$

I take the first order condition with respect to ϕ_t to pin down the optimal portfolio choice.

$$\begin{aligned} &\int_{-\infty}^{\infty} \left\{ \left[\left(\check{r}_{t+1} + 1 - \delta - R_{t+1}\right)^3 + \frac{\sigma^2_{e,t+1}}{2}(1-\gamma)\left[(1-\gamma)\check{r}^2_{t+1} + (1-\delta-R_{t+1})\check{r}_{t+1}\right]\left(\check{r}_{t+1} + 1 - \delta - R_{t+1}\right) \right] \right. \\ &+ \left\{ 2\left(\check{r}_{t+1} + 1 - \delta - R_{t+1}\right)^2 + \frac{\sigma^2_{e,t+1}}{2}\left[(2-3\gamma)\check{r}^2_{t+1} + (2-\gamma)\check{r}_{t+1}(1-\delta-R_{t+1})\right] \right\}(R_{t+1}-1)\phi_t + \\ &\left. + (R_{t+1}-1)^2\left[\left(1 + \frac{\sigma^2_{e,t+1}}{2}\right)\check{r}_{t+1} + 1 - \delta - R_{t+1}\right] \right\} \mathrm{d}F\left(\epsilon^z_{t+1}\right) = 0 \end{aligned} \tag{2.37}$$

The deterministic steady state rules out the aggregate shocks. I list the certainty equivalent and its derivative with respect to ϕ_t at the steady state in the following.

$$
\begin{aligned}
&\mathbb{CE} \\
\approx &\left[\phi\left(\check{r}-\delta\right)+(1-\phi)(R-1)\right]^{1-\gamma}+\frac{1-\gamma}{2}\sigma_e^2\left\{-\gamma\left[\phi\left(\check{r}-\delta\right)+(1-\phi)(R-1)\right]^{-\gamma-1}\left(\phi\check{r}\right)^2+\right. \\
&\left.+\left[\phi\left(\check{r}-\delta\right)+(1-\phi)(R-1)\right]^{-\gamma}\phi\check{r}\right\}^{\frac{1}{1-\gamma}}
\end{aligned}
\tag{2.38}
$$

$$
\begin{aligned}
&\left[\left(\check{r}+1-\delta-R\right)^3+\frac{\sigma_e^2}{2}(1-\gamma)\left[(1-\gamma)\check{r}^2+(1-\delta-R)\check{r}\right]\left(\check{r}+1-\delta-R\right)\right]\phi^2+ \\
&+\left\{2\left(\check{r}+1-\delta-R\right)^2+\frac{\sigma_e^2}{2}\left[(2-3\gamma)\check{r}^2+(2-\gamma)\check{r}(1-\delta-R)\right]\right\}(R-1)\phi+ \\
&+(R-1)^2\left[\left(1+\frac{\sigma_e^2}{2}\right)\check{r}+1-\delta-R\right]=0
\end{aligned}
\tag{2.39}
$$

2.C Coefficient of Variation of Income

I first report the coefficient of variation of entrepreneurs' income at $t+1$ conditional on t's information Coef.Var.$_t(I^i_{t+1})$.

Conditional expectation of $t+1$'s income of entrepreneurs at t is

$$\begin{aligned}
&\mathbb{E}_t(I^i_{t+1})\\
&= \mathbb{E}_t\left[r^i_{t+1}k^i_{t+1} + (R_{t+1}-1)b^i_{t+1}\right]\\
&= \mathbb{E}_t\left\{\nu_t x^i_t\left[\phi_t r^i_{t+1} + (1-\phi_t)(R_{t+1}-1)\right]\right\}\\
&= \nu_t x^i_t\left[\phi_t \alpha z^{\alpha\rho_z}\exp\left(\frac{1}{2}\alpha^2\sigma_z^2\right)K_{t+1}^{\alpha-1} + (1-\phi_t)(R_{t+1}-1)\right];
\end{aligned}$$

Conditional variance of $t+1$'s income of entrepreneurs at t is

$$\begin{aligned}
&\mathrm{Var}_t(I^i_{t+1})\\
&= \mathrm{Var}_t\left[r^i_{t+1}k^i_{t+1} + (R_{t+1}-1)b^i_{t+1}\right]\\
&= \nu_t^2 x_{i,t}^2\mathrm{Var}_t\left[\phi_t r^i_{t+1} + (1-\phi_t)(R_{t+1}-1)\right]\\
&= \nu_t^2 x_{i,t}^2\mathrm{Var}_t\left[\phi_t r^i_{t+1}\right]\\
&= \nu_t^2 x_{i,t}^2\phi_t^2\alpha^2 z^{2\alpha\rho_z}K_{t+1}^{2(\alpha-1)}\left\{\exp\left[\sigma_e^2\exp(-\eta\rho_z\log z_t) + \frac{1}{2}(2\alpha-\eta\sigma_e^2)^2\sigma_z^2\right] - \exp\left(\alpha^2\sigma_z^2\right)\right\}.
\end{aligned}$$

Thus, the coefficient of variation is computed as the dividend of the standard deviation and the expectation.

$$\mathrm{Coef.Var.}_t(I^i_{t+1}) = \frac{\phi_t\alpha z^{\alpha\rho_z}K_{t+1}^{\alpha-1}\sqrt{\exp\left[\sigma_e^2\exp(-\eta\rho_z\log z_t) + \frac{1}{2}(2\alpha-\eta\sigma_e^2)^2\sigma_z^2\right] - \exp\left(\alpha^2\sigma_z^2\right)}}{\phi_t\alpha z^{\alpha\rho_z}\exp\left(\frac{1}{2}\alpha^2\sigma_z^2\right)K_{t+1}^{\alpha-1} + (1-\phi_t)(R_{t+1}-1)}.$$

Next I derive the coefficient of variation of workers' income at $t+1$ conditional on t's information Coef.Var.$_t(I^j_{t+1})$.

Conditional expectation of $t+1$'s income of workers at t is

$$\begin{aligned}
&\mathbb{E}_t(I^j_{t+1}) \\
&= \mathbb{E}_t\left[w_{t+1}e^j_{t+1}\right] \\
&= \mathbb{E}_t\left[(1-\alpha)z^{\alpha}_{t+1}K^{\alpha}_{t+1}e^j_{t+1}\right] \\
&= (1-\alpha)z^{\alpha\rho_z}_t K^{\alpha}_{t+1}e^{\rho_w}_{j,t}\exp\left[\frac{1}{2}(\alpha^2\sigma^2_z+\sigma^2_w)\right];
\end{aligned}$$

Conditional variance of $t+1$'s income of workers at t is

$$\begin{aligned}
&\mathrm{Var}_t(I^j_{t+1}) \\
&= \mathrm{Var}_t\left[w_{t+1}e^j_{t+1}\right] \\
&= \mathrm{Var}_t\left[(1-\alpha)z^{\alpha}_{t+1}K^{\alpha}_{t+1}e^j_{t+1}\right] \\
&= (1-\alpha)^2 z^{2\alpha\rho_z}_t K^{2\alpha}_{t+1}e^{2\rho_w}_{j,t}\left[\exp\left(2\alpha^2\sigma^2_z+2\sigma^2_w\right)-\exp\left(\alpha^2\sigma^2_z+\sigma^2_w\right)\right].
\end{aligned}$$

Then the coefficient of variation is

$$\mathrm{Coef.Var.}_t(I^j_{t+1}) = \sqrt{\exp\left(\alpha^2\sigma^2_z+\sigma^2_w\right)-1}.$$

2.D Welfare for Hand-to-mouth Workers

From (4)

$$\log e^j_{t+1} = \rho_w \log e^j_t + \epsilon^j_{t+1},$$

we have that

$$\mathbb{E}_0\left(\log e_{j,t}\right) = \mathbb{E}_0\left(\rho_w \log e_{j,t-1}\right) + \mathbb{E}_0\left(\epsilon_{j,t}\right) = 0;$$

$$\mathrm{Var}_0\left(\log e_{j,t}\right) = \frac{\sigma_w^2}{1-\rho_w^2};$$

$$\mathbb{E}_0\left(e_{j,t}\right) = \exp\left(\frac{\sigma_w^2}{2(1-\rho_w^2)}\right), \forall t;$$

and furthermore,

$$\mathbb{E}_0\left(e_{j,t}^{1-\gamma}\right) = \exp\left(\frac{\sigma_w^2(1-\gamma)^2}{2(1-\rho_w^2)}\right), \forall t.$$

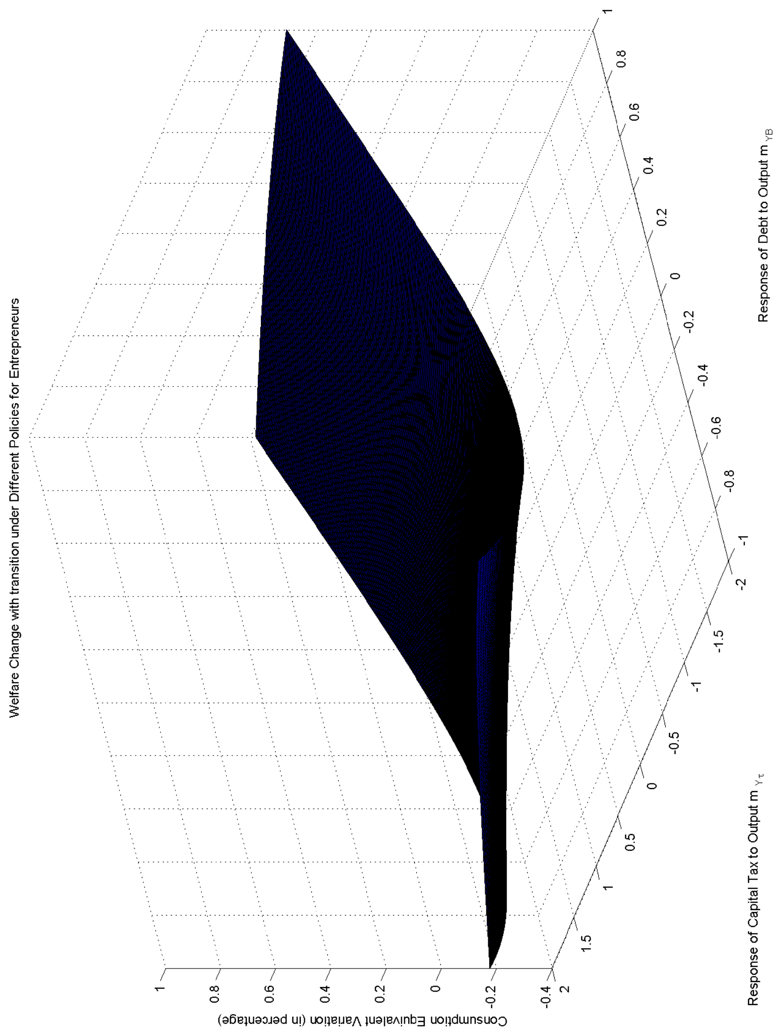
Then the summed unconditional mean of utility at t across workers is

$$\begin{aligned}\int_j \mathbb{E}_0\left\{\frac{(c^j_t)^{1-\gamma}}{1-\gamma}\right\} &= \int_j \mathbb{E}_0\left\{\frac{[(1-\tau^n_t)\frac{1}{\lambda}w_t e_{j,t}]^{1-\gamma}}{1-\gamma}\right\} \\ &= \lambda^\gamma \mathbb{E}_0\left\{\frac{[(1-\tau^n_t)w_t]^{1-\gamma}}{1-\gamma}\right\}\exp\left(\frac{\sigma_w^2(1-\gamma)^2}{2(1-\rho_w^2)}\right).\end{aligned} \tag{2.40}$$

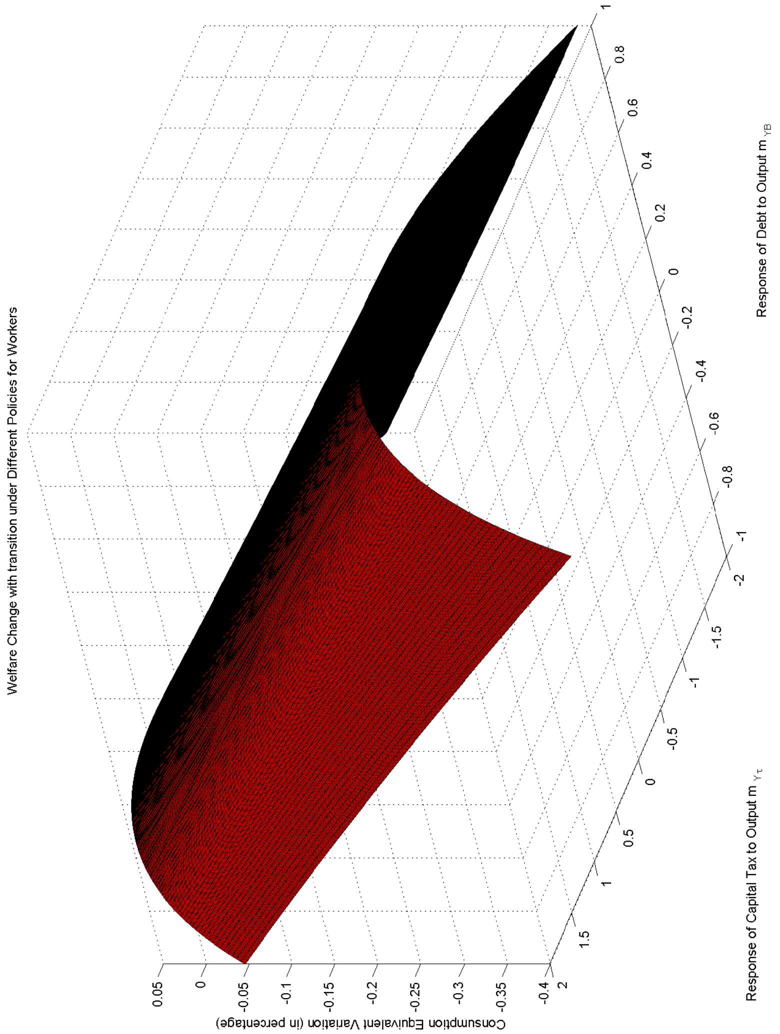
At last, the value function of workers is expressed as

$$\begin{aligned}\int_j V(x^j_0) &= \int_j \mathbb{E}_0 \sum_{t=0}^{\infty}\beta^t \frac{(c^j_t)^{1-\gamma}}{1-\gamma} \\ &= \sum_{t=0}^{\infty}\beta^t \int_j \mathbb{E}_0\left\{\frac{(c^j_t)^{1-\gamma}}{1-\gamma}\right\} \\ &= \lambda^\gamma \sum_{t=0}^{\infty}\beta^t \mathbb{E}_0\left\{\frac{[(1-\tau^n_t)w_t]^{1-\gamma}}{1-\gamma}\right\}\exp\left(\frac{\sigma_w^2(1-\gamma)^2}{2(1-\rho_w^2)}\right).\end{aligned} \tag{2.41}$$

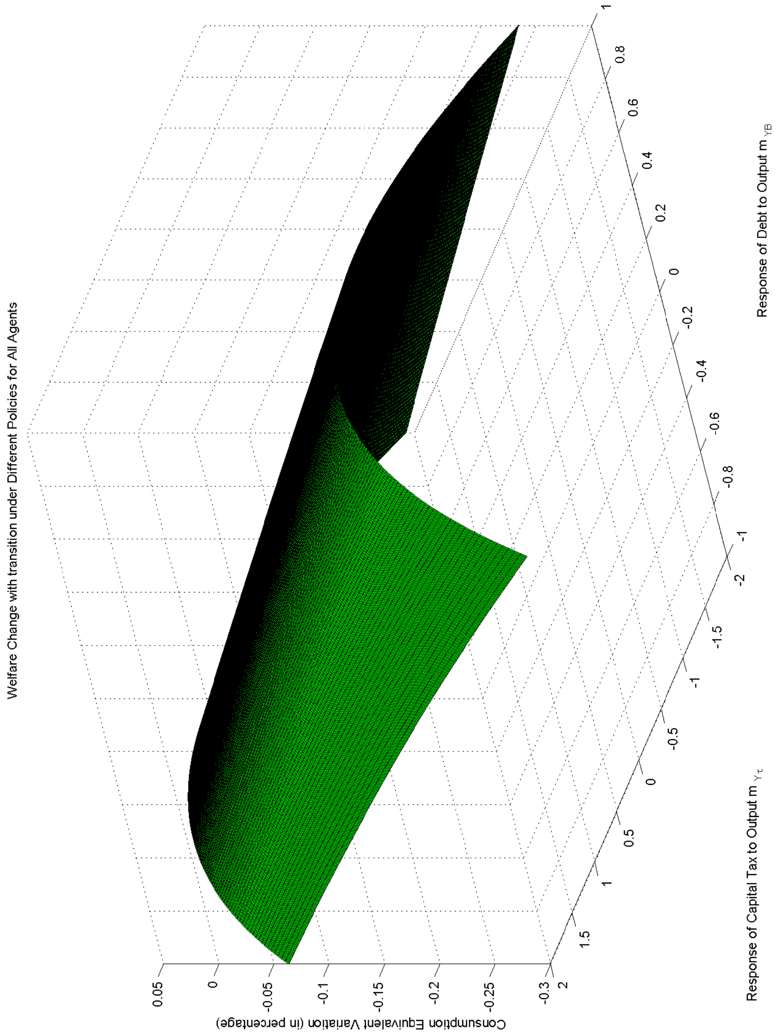
2.E Figure: Welfare Change



A1: Welfare Change Varying Capital Tax and Debt



A1: Welfare Change Varying Capital Tax and Debt (Continued)



A1: Welfare Change Varying Capital Tax and Debt (Continued)

2.F Solution to Modified Entrepreneurs' Problem

$$c_t^i = \frac{\nu_t x_t^i}{1-\tau_t^c}, \tag{2.42}$$

$$k_{t+1}^i = (1-\nu_t)\phi_t x_t^i, \tag{2.43}$$

$$b_{t+1}^i = (1-\nu_t)(1-\phi_t)x_t^i, \tag{2.44}$$

where the marginal propensity to consume out of effective wealth, ν_t, and the share of private equity in the portfolio, ϕ_t, are two stochastic coefficients, depending solely on the current aggregate states, s_t, and satisfying

$$\phi_t = \underset{\phi\in[0,1]}{\arg\max}\, \mathbb{CE}_t \left\{\phi[(1-\tau_{t+1}^k)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] + (1-\phi)R_{t+1}\right\}, \tag{2.45}$$

$$\frac{\nu_t^{-\gamma}}{(1-\tau_t^c)^{1-\gamma}} = \beta_s(1-\nu_t)^{-\gamma}\mathbb{E}_t\left\{\frac{\nu_{t+1}^{-\gamma}}{(1-\tau_{t+1}^c)^{1-\gamma}}\{\phi_t[(1-\tau_{t+1}^k)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] + (1-\phi_t)R_{t+1}\}^{1-\gamma}\right\} \tag{2.46}$$

where $\mathbb{CE}$ represents the certainty equivalent of an entrepreneur.[5]

Define the value function for entrepreneurs as $V(x_t^i)$ which is given by

$$V(x_t^i) = \frac{\nu_t^{-\gamma}(x_t^i)^{1-\gamma}}{(1-\gamma)(1-\tau_{t+1}^c)^{1-\gamma}}. \tag{2.47}$$

[5] I denote $\beta_s = \beta(1-Pr_d)$

Table 2.5: Welfare Comparison of Fiscal Policy (in Percentage)

Variables	**Policy Maximizing Entrepreneurs' Welfare** $m_{Y\tau} = -2$, $m_{YB} = 1$		**Policy Maximizing Workers' Welfare** $m_{Y\tau} = 2$, $m_{YB} = -0.54$		**Policy Maximizing Social Welfare** $m_{Y\tau} = -2$, $m_{YB} = -0.37$	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**
Bond	5.69	14.81	-5.47	-22.96	-0.16	-39.33
Cap. Tax	-0.20	1.45	0.44	1.78	-0.35	1.95
Capital	0.44	-42.78	-0.63	-24.88	0.44	5.62
Entr. Con.	1.32	169.70	-1.58	-23.24	0.50	60.65
Wage	0.16	-24.67	-0.22	-8.34	0.14	0.24
Lab. Tax	0.32	0.45	-0.37	-0.07	0.11	-0.15
Wor. Con.	-0.19	10.33	0.23	0.32	-0.03	-2.90
Welfare Change (Consumption Equivalent Variation)						
Entrepreneur	0.705		-0.321		0.239	
Worker	-0.385		0.044		-0.022	
Total	0.014		-0.006		0.235	

The values for all variables are percentage change compared to the baseline policy combination ($m_{Y\tau} = 0.91$, $m_{YB} = -0.62$). Cap. Tax and Lab. Tax denote the capital tax rate and the labor tax rate, respectively. Entr. Con., Wor. Con., and Ttl. Con. represent consumption for entrepreneurs, consumption for workers and consumption for total agents, respectively.

Bibliography

[1] Ahrens, S., I. Pirschel, and D. J. Snower. A Theory of Price Adjustment under Loss Aversion. *Journal of Economic Behavior and Organization*, 134:78–95, 2017.

[2] Aiyagari, S. R. Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics*, 109(3):659–684, 1994.

[3] Aiyagari, S. R. and E. R. Mcgrattan. The Optimum Quantity of Debt. *Journal of Monetary Economics*, (3):447–469, 1998.

[4] Aiyagari, S. R., A. Marcet, T. J. Sargent, and J. Seppälä. Optimal Taxation without State-Contingent Debt. *Journal of Political Economy*, 110(6):1220–1254, 2002.

[5] Andries, M. Consumption-Based Asset Pricing with Loss Aversion. *Working Paper*, 2012.

[6] Ang, A., J. Chen, and Y. Xing. Downside Risk. *Review of Financial Studies*, 19(4): 1191–1239, 2006.

[7] Angeletos, G.-M. Uninsured Idiosyncratic Investment Risk and Aggregate Saving. *Review of Economic Dynamics*, 10(1):1–30, January 2007.

[8] Angeletos, G.-M. Uninsured Idiosyncratic Investment Risk and Aggregate Saving. *Review of Economic Dynamics*, 10(1):1–30, January 2007.

[9] Angeletos, G.-M. and L. E. Calvet. Idiosyncratic Production Risk, Growth and the Business Cycle. *Journal of Monetary Economics*, 53(6):1095–1115, 2006.

[10] Angeletos, G.-M. and L. E. Calvet. Idiosyncratic Production Risk, Growth and the Business Cycle. *Journal of Monetary Economics*, 53(6):1095–1115, 2006.

[11] Angeletos, G.-M. and V. Panousi. Revisiting the Supply Side Effects of Government Spending. *Journal of Monetary Economics*, 56(2):137–153, March 2009.

[12] Angeletos, G.-M. and V. Panousi. Financial Integration, Entrepreneurial Risk and Global Dynamics. *Journal of Economic Theory*, 146(3):863–896, 2011.

[13] Arseneau, D. M. and S. K. Chugh. Tax Smoothing in Frictional Labor Markets. *Journal of Political Economy*, 120(5):926–985, 2012.

[14] Barberis, N. and M. Huang. Mental Accounting, Loss Aversion, and Individual Stock Returns. *The Journal of Finance*, 56(4):1247–1292, 2001.

[15] Barberis, N., M. Huang, and T. Santos. Prospect Theory and Asset Prices. *The Quarterly Journal of Economics*, 116(1):1–53, 2001.

[16] Bassetto, M. Optimal Fiscal Policy with Heterogeneous Agents. *Quantitative Economics*, 5(3):675–704, 2014.

[17] Bassetto, M. and J. Benhabib. Redistribution, Taxes, and the Median voter. *Review of Economic Dynamics*, 9(2):211–223, 2006.

[18] Benartzi, S. and R. H. Thaler. Myopic Loss Aversion and the Equity Premium Puzzle. *The Quarterly Journal of Economics*, 110(1):73–92, 1995.

[19] Benhabib, J., A. Bisin, and S. Zhu. The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents. *Econometrica*, 79(1):123–157, 2011.

[20] Berkelaar, A. and R. Kouwenberg. From Boom 'til Bust: How Loss Aversion Affects Asset Prices. *Journal of Banking and Finance*, 33(6):1005–1013, 2009.

[21] Bianchi, J. and E. G. Mendoza. Optimal Time-Consistent Macroprudential Policy. *Working Paper*, 2013.

[22] Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry. Really Uncertain Business Cycles. December 2016.

[23] Cagetti, M. and M. De Nardi. Entrepreneurship, Frictions, and Wealth. *Journal of Political Economy*, 114(5):835–870, 2006.

[24] Cagetti, M. and M. De Nardi. Estate Taxation, Entrepreneurship, and Wealth. *The American Economic Review*, 99(1):85–111, 2009.

[25] Camerer, C., L. Babcock, G. Loewenstein, and R. Thaler. Labor Supply of New York City Cabdrivers: One Day at a Time. *The Quarterly Journal of Economics*, 112(2): 407–441, 1997.

[26] Carroll, C. D. Portfolios of the Rich. *In: Guiso, L., Haliassos, M., Jappelli, T. (Eds.), Household Portfolios. MIT Press, Cambridge,MA*, 2001.

[27] Castañeda, A., J. Díaz-Giménez, and J.-V. Rios-Rull. Exploring the Income Distribution Business Cycle Dynamics. *Journal of Monetary Economics*, 42(1):93–130, 1998.

[28] Castañeda, A., J. Díaz-Giménez, and J.-V. Rios-Rull. Accounting for the U.S. Earnings and Wealth Inequality. *Journal of Political Economy*, 111(4):818–857, 2003.

[29] Chamley, C. Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. *Econometrica*, pages 607–622, 1986.

[30] Chari, V. V., L. J. Christiano, and P. J. Kehoe. Optimal Fiscal Policy in a Business Cycle Model. *Journal of Political Economy*, 102(4):617–652, 1994.

[31] Chari, V. V., L. J. Christiano, and P. J. Kehoe. Optimal Fiscal Policy in a Business Cycle Model. *Journal of Political Economy*, 102(4):617–652, 1994.

[32] Chugh, S. K. Optimal Fiscal and Monetary Policy with Sticky Wages and Sticky Prices. *Review of Economic Dynamics*, 9(4):683–714, 2006.

[33] Chugh, S. K. Optimal Inflation Persistence: Ramsey Taxation with Capital and Habits. *Journal of Monetary Economics*, 54(6):1809–1836, 2007.

[34] Coeurdacier, N., H. Rey, and P. Winant. The Risky Steady State. *The American Economic Review*, 101(3):398–401, May 2011.

[35] Conesa, J. C., S. Kitao, and D. Krueger. Taxing Capital? Not a Bad Idea After All! *The American Economic Review*, 99(1):25–48, 2009.

[36] Covas, F. Uninsured Idiosyncratic Production Risk with Borrowing Constraints. *Journal of Economic Dynamics and Control*, 30(11):2167–2190, 2006.

[37] Croce, M. M., H. Kung, T. T. Nguyen, and L. Schmid. Fiscal Policies and Asset Prices. *Review of Financial Studies*, 25(9):2635–2672, 2012.

[38] Davila, J., J. H. Hong, P. Krusell, and J.-V. Ríos-Rull. Constrained Efficiency in the Neoclassical Growth Model with Uninsurable Idiosyncratic Shocks. *Econometrica*, 80(6):2431–2467, 2012.

[39] DeBacker, J. M., B. T. Heim, V. Panousi, S. Ramnath, and I. Vidangos. The Properties of Income Risk in Privately Held Businesses. *Working Paper*, 2012.

[40] Devereux, M. P., B. Lockwood, and M. Redoano. Do Countries Compete over Corporate Tax Rates? *Journal of Public Economics*, 92(5):1210–1235, 2008.

[41] Díaz-Giménez, J. and J. Pijoan-Mas. Flat Tax Reforms: Investment Expensing and Progressivity. 2011.

[42] Dimmock, S. G. and R. Kouwenberg. Loss-Aversion and Household Portfolio Choice. *Journal of Empirical Finance*, 17(3):441–459, 2010.

[43] Goldberg, J. E. Idiosyncratic Investment Risk and Business Cycles. 2014.

[44] Guner, N., R. Kaygusuz, and G. Ventura. Income Taxation of U.S. Households: Facts and Parametric Estimates. *Review of Economic Dynamics*, 17(4):559–581, 2014.

[45] Guvenen, F., S. Ozkan, and J. Song. The Nature of Countercyclical Income Risk. *Journal of Political Economy*, 122(3):621–660, 2014.

[46] Heathcote, J., K. Storesletten, and G. L. Violante. Quantitative Macroeconomics with Heterogeneous Households. *Annual Review of Economics*, 1(1):319–354, 2009.

[47] Heathcote, J., F. Perri, and G. L. Violante. Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006. *Review of Economic Dynamics*, 13(1):15–51, 2010.

[48] Hintermaier, T. and W. Koeniger. On the Evolution of the US Consumer Wealth Distribution. *Review of Economic Dynamics*, 14(2):317–338, 2011.

[49] Judd, K. L. Redistributive Taxation in a Simple Perfect Foresight Model. *Journal of Public Economics*, 28(1):59–83, 1985.

[50] Kahneman, D. and A. Tversky. Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, pages 263–291, 1979.

[51] Kaplan, G., G. L. Violante, and J. Weidner. The Wealthy Hand-to-mouth. *Working Paper*, 2014.

[52] Karantounias, A. G. Managing Pessimistic Expectations and Fiscal Policy. *Theoretical Economics*, 8(1):193–231, 2013.

[53] Kőszegi, B. and M. Rabin. Reference-Dependent Consumption Plans. *The American Economic Review*, 99(3):909–936, 2009.

[54] Krueger, D., K. Mitman, and F. Perri. Macroeconomics and Household Heterogeneity. *Handbook of Macroeconomics*, 2:843–921, 2016.

[55] Krusell, P. and A. A. Smith, Jr. Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, 106(5):867–896, 1998.

[56] Leeper, E. M. Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies. *Journal of Monetary Economics*, 27(1):129–147, 1991.

[57] Lepetyuk, V. and C. A. Stoltenberg. Reconciling Consumption Inequality with Income Inequality. *Working Paper*, 2013.

[58] Lucas, R. E. *Models of Business Cycles*, volume 26. Basil Blackwell Oxford, 1987.

[59] Lucas, R. E. and N. L. Stokey. Optimal Fiscal and Monetary Policy in an Economy without Capital. *Journal of Monetary Economics*, 12(1):55–93, 1983.

[60] Meh, C. A. Business Risk, Credit Constraints, and Corporate Taxation. *Journal of Economic Dynamics and Control*, 32(9):2971–3008, 2008.

[61] Meh, C. A. and V. Quadrini. Endogenous Market Incompleteness with Investment Risks. *Journal of Economic Dynamics and Control*, 30(11):2143–2165, 2006.

[62] Moskowitz, T. J. and A. Vissing-Jørgensen. The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle? *The American Economic Review*, 92(4): 745–778, 2002.

[63] Nezafat, P. and C. Slavik. Asset Prices and Business Cycles with Financial Shocks. *Working Paper*, 2015.

[64] Nirei, M. and S. Aoki. Pareto Distribution of Income in Neoclassical Growth Models. *Review of Economic Dynamics*, 20:25–42, 2016.

[65] O'Connell, P. G. and M. Teo. Institutional Investors, Past Performance, and Dynamic Loss Aversion. *Journal of Financial and Quantitative Analysis*, 44(1):155–188, 2009.

[66] Pagel, M. Expectations-Based Reference-Dependent Life-Cycle Consumption. *Working Paper*, 2013.

[67] Pagel, M. Expectations-Based Reference-Dependent Preferences and Asset Pricing. *Journal of the European Economic Association*, 2015.

[68] Panousi, V. Capital Taxation with Entrepreneurial Risk. *Working Paper*, 2010.

[69] Pope, D. G. and M. E. Schweitzer. Is Tiger Woods Loss Averse? Persistent Bias in the Face of Experience, Competition, and High Stakes. *The American Economic Review*, 101(1):129–157, 2011.

[70] Quadrini, V. Entrepreneurship in Macroeconomics. *Annals of Finance*, 5(3):295–311, 2009.

[71] Schmitt-Grohé, S. and M. Uribe. Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function. *Journal of Economic Dynamics and Control*, 28(4):755–775, 2004.

[72] Schmitt-Grohé, S. and M. Uribe. Optimal Fiscal and Monetary Policy under Imperfect Competition. *Journal of Macroeconomics*, 26(2):183–209, 2004.

[73] Schmitt-Grohé, S. and M. Uribe. Optimal Fiscal and Monetary Policy under Sticky Prices. *Journal of Economic Theory*, 114(2):198–230, 2004.

[74] Schmitt-Grohé, S. and M. Uribe. An OLS Approach to Computing Ramsey Equilibria in Medium-Scale Macroeconomic Models. *Economics Letters*, 115(1):128–129, 2012.

[75] Storesletten, K., C. I. Telmer, and A. Yaron. Consumption and Risk Sharing over the Life Cycle. *Journal of Monetary Economics*, 51(3):609–633, 2004.

[76] Tversky, A. and D. Kahneman. Loss Aversion in Riskless Choice: A Reference-Dependent Model. *The Quarterly Journal of Economics*, 106(4):1039–1061, 1991.

[77] Tversky, A. and D. Kahneman. Advances in Prospect Theory: Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty*, 5(4):297–323, 1992.

[78] Werning, I. Optimal Fiscal Policy with Redistribution. *The Quarterly Journal of Economics*, 122(3):925–967, 2007.